Applied Education Research

Knowledge and Training of Secondary School Mathematics Teachers

Status Report and Recommendations

"The Knowledge Base for Teaching Mathematics" Committee

Editors: Hanokh Gutfreund and Yehoshua Rosenberg

The Initiative for Applied Education Research
The Israel Academy of Sciences and Humanities

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Jerusalem, 2012

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The Initiative for Applied Education Research provides Israeli decision-makers in the field of education with the most up-to-date, scientific, critically-appraised knowledge, so they can better formulate policy and design interventions that improve achievements in Israeli education.

The Initiative's vision: Research knowledge is an essential component in the planning of public policy or large-scale interventions. Critically-appraised research knowledge supports the formulation of policy that has a greater chance of success and is more apt to promote rational public discourse. The Initiative implements this vision in the field of education.

The Initiative's modus operandi: Generally speaking, the issues the **Initiative** addresses are first raised by decision-makers. Following consultations with senior Ministry of Education officials and other stakeholders, the **Initiative**'s steering committee, appointed by the president of the Israel Academy of Sciences and Humanities, oversees both the creation of a work plan and the peer-review process that precedes publication of produced reports.

The **Initiative** sets up expert committees and convenes symposia for researchers, education professionals and decision-makers. It publishes reports of its work and makes them readily available to the public. Those serving on its expert committees do so without remuneration.

History of the Initiative: The **Initiative** was established in 2003 as a joint venture between the Israel Academy of Sciences and Humanities, the Israel Ministry of Education, and the Rothschild Foundation (Yad Hanadiv), which initiated the project. Yad Hanadiv generously provided all the necessary funding in the early days until 2010, when the **Initiative** began operating as a unit of the Israel Academy of Sciences and Humanities.

In the summer of 2010, the Israeli Knesset amended the Israel Academy of Sciences and Humanities Law, regulating the Israel Academy's advisory role vis-à-vis government ministries seeking its services.

The Initiative for Applied Education Research is now responsible **for all the consulting services on education-related issues provided by the Israel Academy to the Israeli government and various authorities.**

The "Knowledge Base for Teaching Secondary School Mathematics" Committee

At the Ministry of Education's behest, the Israel Academy of Sciences and Humanities established an expert committee to address the issue of the "Knowledge-Base for Teaching Secondary School Mathematics." The committee operated under the auspices of the Initiative for Applied Education Research, a unit of the Israel Academy.[1] The committee's aim was to recommend policy, on the basis of analysis of research conducted in Israel and abroad and based on the experience of professionals in Israel and other countries, which would advance mathematics education in Israel and improve student achievement.

The committee, chaired by Professor (Emeritus) Hanokh Gutfreund of the Hebrew University of Jerusalem, began its work in the spring of 2010. The committee held deliberations, heard information and opinions from a variety of experts, commissioned status reviews and scientific reviews and held symposia open to the public. When its deliberations came to a close, the committee drafted a summarizing document, approved by all its members, that includes, among other things, policy recommendations geared to improving student achievement. Following peer review, the document was submitted to Ministry of Education officials and in the spring of 2012, was made available to the public on the Initiative's website.

Committee members:

Prof. Hanokh Gutfreund, Chair (Hebrew University of Jerusalem)
Prof. Abraham Arcavi (Weizmann Institute of Science)
Dr. Meir Buzaglo (Hebrew University of Jerusalem)
Prof. Ruhama Even (Weizmann Institute of Science)
Dr. Hagar Gal (David Yellin College of Education)
Dr. Bella Kessler (Hebrew University of Jerusalem High School)

[1] Yad Hanadiv also supported the arrangement between the Ministry of Education and the Israel Academy.

Prof. Raz Kupferman (Hebrew University of Jerusalem)
Prof. Roza Leikin (University of Haifa)
Prof. Varda Liberman (Interdisciplinary Center Herzliya)
Ms. Ifat Nachshon (Nehalim Torah Technology High School and Davidson Institute for Science Education)
Prof. Fadia Nasser-Abu Alhija (Tel Aviv University)
Prof. Anna Sfard (University of Haifa)

Study director: Yehoshua Rosenberg

Acknowledgements

During the two years of its activity, the “Knowledge Base for Teaching Secondary-School Mathematics” Committee was aided by many experts from the fields of research and practice. Now that its work has come to a close and the report is being published, the committee would like to thank all those who contributed of their time, expertise and good will.

First and foremost, the committee members wish to thank the Ministry of Education (MOE) officials who readily responded to our request to meet and provide both written and oral data and information, answered our questions and shared their experience with us.[2] To Dr. Gaby Avital, former chief scientist, Dr. David Feilchenfeld, nationwide adviser for secondary school mathematics instruction at the MOE, Dr. Irma Jan, chief inspector for mathematics, Prof. Zecharia Madar, chief scientist at the MOE, Dr. Asaad Mahajne, inspector for mathematics instruction in the Arab sector, Mr. Moti Rosner, director of Division A for teacher professional development at the MOE, Dr. Sliman Salamy, inspector for mathematics in the Druze and Circassian sector. Special appreciation to Dr. Hanna Perl, head of the mathematics and sciences section and Prof. Azriel Levy, chair of the mathematics profession committee who accompanied the committee at all stages of its work.

The committee was assisted by experts in research and teaching who shared their knowledge, experience and wisdom with the members: Prof. Deborah Loewenberg-Ball of the University of Michigan in the U.S. visited Israel and met with committee representatives on two occasions and brought them up to date with important and recent information. Prof. Michele Artigue of the University Paris 7 was the committee’s guest in Israel and shared her great knowledge and experience with its members. Prof. Magdalene Lampert of the University of Michigan in the U.S. met with committee members and contributed of her experience.

[2] Names are listed in alphabetical order by surname. The position or place of work mentioned is correct for the period in which the meetings took place.

The committee would like to thank all the experts who agreed to lecture before it in various forums: Prof. Ron Aharoni of the Technion in Haifa, Prof. Zvi Artstein of the Weizmann Institute of Science, Prof. Michal Beller, director general of the National Authority for Measurement and Evaluation in Education, Prof. Miriam Ben-Peretz of the University of Haifa, Prof. Yair Caro of the Oranim College of Education, Dr. Naomi Chissick, director of mathematics education at the ORT network, Prof. Nitza Cohen of the David Yellin College of Education, Prof. Ehud De Shalit of the Hebrew University of Jerusalem, Mr. Yoel Geva, director of the "Yoel Geva" network of schools, author of mathematics textbooks, Dr. Orly Gottlib of the "Orot Israel" College of Education in Elkana, Prof. Avishai Henik of Ben-Gurion University of the Negev, Dr. Ronnie Karsenty of the Weizmann Institute of Science, Prof. Talma Leviatan of Beit Berl College, Prof. Nitsa Movshovitz-Hadar of the Technion in Haifa, Dr. Tahl Nowik of Bar Ilan University, Ms. Ruth Ottolenghi, former head of the MOE's secondary education division, Prof. (Emeritus) Levy Rahmani of Tel Aviv University, Dr. Orly Rubinstein of the University of Haifa, Dr. Atara Shriki of the Oranim College of Education, Prof. (Emeritus) Ruth Stavy of Tel Aviv University, Prof. Joseph Tzelgov of Ben-Gurion University of the Negev, and Prof. Shlomo Vinner of Ben-Gurion University of the Negev.

Special thanks to David Maagan, head of the higher education and teaching force section in the chief scientist's office at the Central Bureau of Statistics, who, along with his staff, eagerly undertook to help the committee and placed at its disposal the finest data and analyses on the topic of secondary school mathematics teachers.

In the course of its work, the committee issued calls for proposals and commissioned scientific reviews on selected topics central to its work. The committee would like to thank the researchers for writing the reviews which served as a sound foundation for its work: Dr. Iddo Gal, Dr. Sarit Gariba, Dr. Einav Aizikovitsh-Udi, Isaac Slater and Carmel Shor.

The committee also thanks the teachers and experts who responded to its call and gave of their knowledge and experience: Yael Adamovsky, Dr. Yehuda Ashkenazi, Dr. Tamara Avissar, Besan Awsaf, Meir Bechor, Dr.

Michael Belousov, Dr. Sarah Black, Moshe Cocos, Elisheva Cohen, Dr. Joseph Dalin, Sara Grazotis, Amos Guetta, Dr. Ronit Hoffman, Maya Koren, Bracha Levy and Luba Zahavi, Michal Madmon, Batya Naiman, Omri Nave, Michael Rozenberg, Alon Schejter and Dr. Philip Slobodsky.

The members especially thank the team of teachers that worked alongside the committee: Rivka Maimon (ORT-Sapir, Yerucham), Sariga Dinur (City High School C, Haifa), Dudi Meidzinski (Manor-Kabri, Kibbutz Kabri), Afnan Yahia (the Kfar Kara High School), Mira Kahana (Ostrovsky High School, Raanana), and Rina Shitrit (AMIT School, Beersheba).

Sincere thanks to Dr. Varda Talmon and the staff of the National Center for Secondary School Mathematics Teachers for their fruitful and pleasant cooperation in the annual convention of math teachers and for the discourse which has since ensued.

The committee is grateful to the staff of the Israel Academy of Sciences and Humanities for helping to fulfill our every request and responding to the committee's needs throughout its work, and to Yad Hanadiv which enabled the committee's activity.

Immeasurable appreciation to the staff of the Initiative for Applied Education Research for their attention and support throughout: Thanks to Ziva Dekel, Riki Fishel, Udit Nisan, Liat Netzer, Reut Issachar, Avia Shemesh, Itay Pollak and Devora Fried. A huge garland of thanks to the Initiative's Ada Paldor, responsible for the linguistic editing of this document's chapters, who did her work with wisdom, professionalism and much patience. Special appreciation to Dr. Avital Darmon, director of the Initiative, who closely, faithfully and consistently followed the committee's work, and from the very beginning through to publication of this summarizing document, aided it in thinking through and processing the many issues with which it dealt.

A draft of the document summarizing the committee's work was submitted for peer review in Israel and abroad. The identity of the judges was not known to the committee members until the time of publication. The peer review process is designed to ensure an external, professional critique, to the point and impartial, which will help the report's authors improve their work and turn it into a document that will be used. The committee thanks the reviewers for reading the document and writing their critiques.

The names of the reviewers are (in alphabetical order):

Prof. (Emeritus) Dan Amir, School of Mathematical Sciences, Tel Aviv University

Prof. Guershon Harel, Department of Mathematics, University of California, San Diego, (U.S.)

Ms. Haya Shitay, head of the Education Administration in the Modiin, Maccabim-Reut Municipality

Prof. (Emeritus) Shlomo Vinner, Department of Science and Mathematics Teaching, Hebrew University of Jerusalem and Ben-Gurion University of the Negev

The above-mentioned reviewers made constructive comments and suggested additions and revisions to the draft they read. At the same time, they were not asked to adopt the committee's conclusions or its recommendations, nor did they see the revised version of the document before it was sent to print. Responsibility for the document's final content rests entirely with the expert committee.

With gratitude and appreciation to all.

Hanokh Gutfreund, Chair   Yehoshua Rosenberg, Coordinator

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Chapter 1
Preface

1.1 Establishing the committee and its work methods

In an effort to advance mathematics education in Israel, Ministry of Education officials turned to the Israel Academy of Sciences and Humanities with a request to establish an expert committee that would formulate recommendations regarding the knowledge base required by secondary school mathematics teachers. The broad scope of the mathematics education system and the high cost of implementing of any change require a meticulous examination of the subject. The committee was charged with the tasks of compiling an overview of current relevant academic research from Israel and abroad, of learning from the experience of professionals in the field, of analyzing the components of different kinds of knowledge and the degree to which they are necessary, and finally of reaching joint conclusions and recommending policy with the potential for improving student achievement.

As a division of the Israel Academy of Sciences and Humanities, the role of the Initiative for Applied Education Research ("The Initiative") is to provide education system decision makers with up-to-date, evidence-based knowledge to assist them in their work and improve the education system's achievements. In response to the Ministry of Education's request, the Initiative established an expert committee, comprised of researchers from various fields of expertise and teachers of mathematics, to examine the knowledge base required by secondary school mathematics teachers.

The committee was established in the wake of growing concern, shared by educators, education policy makers and the public at large, regarding the status of mathematics education in Israel, reflected in Israeli students' low achievements on international tests such at the TIMMS and PISA. The data also indicates both a shortage of math teachers and a decrease in the number of those who choose to pursue such training. Many teachers who arrived during the wave of immigration from the former Soviet Union are now approaching retirement age, and the number of prospective teachers is inadequate for current needs. Only a minority of teachers meets the Ministry of Education's requirement for a master's degree and principals of high schools find themselves assigning math classes to teachers of other fields of specialty. This is done out of necessity but stems from a basic assumption that anyone who possesses some mathematical knowledge is capable of teaching math on a secondary school level. In this context, there are those who speak of a crisis that will exacerbate if appropriate steps are not taken to improve the situation.

The committee, which operates without remuneration, began its work in the spring of 2010. It met with experts from Israel and abroad, academicians, educators and various interested parties; it commissioned scientific reviews on diverse topics related to teaching math, and held symposia open to the public.

The committee decided in advance on the topics to be addressed and the issues for which recommendations would be formulated. Its primary mandate was to define the essential and the desirable components of the intellectual and professional universe of secondary school mathematics teachers. The committee extended its mandate to include the issue of teacher training – including the desired background and competencies. This was deemed necessary in light of the shortage of suitable personnel and the pressing need to develop appropriate training programs. The committee formulated recommendations on academic frameworks for teacher training, duration of training and the role of institutions of higher learning in this effort. It also dealt with controls placed on entering the profession via licensing and certification exams. It devoted a portion of its deliberations to the system of in-service education and support frameworks that would guarantee the professional development of secondary school math teachers throughout their careers. The committee issued an appeal to math teachers to express their opinions on these topics. A team of teachers that had, on occasion, worked with the committee then presented the submitted responses, together with committee members, at a convention of math teachers at Shefayim.

The committee viewed the role of the teacher as the most influential school-related factor affecting student achievement. At the same time, it was clear to the committee members that there are additional factors contributing to how teachers advance their students. These relate to their physical environment and the tools available to teachers in performing their work. The committee also briefly addressed this aspect of the problem. Its conclusions regarding professional development and quality control are not specific to teaching mathematics; they are relevant to other fields of teaching as well. However, in keeping with its mandate, these issues were deliberated only in connection with their implications for secondary school mathematics teaching.

Much research has been carried out on the topics the committee addressed, but existing data does not point to clear policy preferences in all of these areas. Where the data was not categorical, the committee relied on the collective insights, experience and intuition its members acquired in the course of their work. The committee operated on a consensual basis and all of its members bear responsibility for the final document; the writing process involved collaboration among the members who participated in composing the various chapters. After all the members had contributed their comments and suggestions, a final version was drafted and authorized by the committee as a whole.

The committee members believe that adopting and implementing the recommendations contained in the present report can lead to an improvement in mathematics instruction and in student achievement, increase interest in the subject of mathematics among students, and raise the prestige and status of those engaged in teaching math in their own eyes as well as in the public's perception.

Implementing the report's recommendations requires consent, support and vigorous activity on the part of all the entities and institutions that comprise the systems of education and higher learning in Israel – the Ministry of Education, academic institutions and especially, the departments of mathematics and departments of education at these institutions, the Council for Higher Education, the Planning and Budgeting Committee (VATAT) and teachers' organizations.

1.2 Summary of main recommendations

We wish to present our main recommendations at the outset. The background to these recommendations, their details and additional recommendations, related to and derived from them, appear in the relevant chapters of this report.

1. Knowledge Components

The committee recommends that the curricula in all training and professional development programs for secondary school teachers of mathematics be based on the following knowledge components:

★ **General mathematics knowledge at the level of a bachelor's degree in math, mathematics knowledge related to teaching and mathematics knowledge in different contexts including an emphasis on the history of mathematics, mathematical literacy and the use of mathematics in various sciences**

★ General pedagogical knowledge and pedagogical knowledge particular to teaching math

★ Practical knowledge concerning the daily work of the teacher in the classroom

These components are detailed in Chapter 3.

2. Training Program

The committee recommends that institutions of higher learning develop a special four-year program leading to a master's degree in mathematics education.

Participants in this track will receive a bachelor's degree after the third year and upon completion of the four-year track, a teaching certificate as well. **The committee views he proposed program as the preferred track for training secondary school math teachers and hopes that in the future this will be the primary training program.**

Students accepted into this program will be awarded generous scholarships during their period of study and following their year of student teaching, graduates will commit to teaching for a number of years within the education system.

Details and additional recommendations can be found in Chapter 4, section 4.5.

3. Licensing Examination

The committee is aware of the existing shortage of math teachers for secondary schools and of the heterogeneity of the different training programs for math teachers. However, it still views the existence of a knowledge base common to graduates of all programs as being of the utmost importance. **Thus, it recommends instituting a licensing examination to validate the existence of the required knowledge.** Furthermore, the committee believes that an examination of this kind will grant mathematics teaching the professional status and prestige a licensed field confers.

Details and additional recommendations can be found in Chapter 5, section 5.5.

4. Professional Development

The committee attaches great importance to mathematics teachers' professional development and recommends that it be viewed as an integral and essential part of the teacher's job. To this end, effective systems of professional development must be developed within schools as well as in other settings, and professional standards must be set to which they will adhere.

Details and additional recommendations can be found in Chapter 6, section 6.5.

5. Teacher Educators

The committee recommends opening a course of training for teacher educators of secondary school math teachers at either the certificate

or master's degree level at institutions of higher education, and likewise recommends establishing the role of teacher educators as a profession.

Details and additional recommendations can be found in Chapter 7, section 7.5.

6. Teacher Evaluation

The committee recommends holding formative and summative evaluation frameworks for math teachers at different stages of their work. The evaluations will be performed by the teachers themselves, teacher educators, and experts in math and math education.

Details and additional recommendations can be found in Chapter 8, section 8.6.

7. Teaching Environment

The committee recommends investing in enhancing the teaching environment and allocating appropriate resources to support math teachers' work, those that will facilitate the expression of their knowledge and abilities.

Details and additional recommendations can be found in Chapter 9.

* The following recommendation is a minority view presented in the name of Dr. Hagar Gal. The majority of committee members believe that a recommendation of this nature is beyond the committee's scope and are not a party to it:

> In light of the demand for a bachelor's degree in mathematics, teacher training institutions, including teaching colleges that have demonstrated their ability to meet the required academic level in the field, should be permitted to grant a bachelor's degree.

Chapter 2
Introduction

2.1 Background

Mathematics, like other subjects studied in secondary school, is important in shaping, fostering and building the identity, intellect, education and sound functioning of every citizen, helping them to contribute to themselves and to the society in which they live. However, the committee is aware of the special status the general public, the media and the system of higher education attributes, directly or indirectly, to the subject of mathematics.

The special status of mathematics stems from its many uses in other fields, from the fact that it is defined – justifiably or not – as a prerequisite for further study in both math-intensive and other fields, and from the image accorded to those who succeed in mathematics. Success in mathematics not only opens opportunities for further study or gainful employment, it also contributes to the formation of a positive self-image. These characteristics are less closely associated with other subjects.

The special status of mathematics also derives from the inherent nature of the subject, such as precision in definitions, economy of words, pursuit of abstraction and simplicity, use of symbolic and powerful language, and, in particular, the fundamental need to prove assertions. Many fields seek to prove assertions, but only a mathematical text demands the full meaning of the concept of proof. That is, every assertion must derive from a previous assertion based on the strict rules of logical deduction. Mathematics establishes a culture of rational thinking that seeks to resist any unjustified acceptance of assertions (or opinions).

The concern for the level of education and knowledge in the State of Israel spans subjects and fields of content. However, due to the factors described above, the state of the study of mathematics has aroused particular concern

The concern for the level of education and knowledge in the State of Israel spans subjects and fields of content. However, the state of the study of mathematics has aroused particular concern in recent years.

in recent years – concern that is shared by teachers, education policy makers and the public at large. This concern also finds broad expression in the media, which highlights the various problematic aspects of math education in Israel – low achievements on national and international exams, a shortage of teachers with suitable training, a decrease in the number of students choosing math-intensive fields in the universities, deficiencies in the background of those studying math-intensive subjects, and so on.

A look at the system of math education raises a considerable number of questions that should be addressed at the level of defining objectives and at the level of systematically collecting facts as a basis for sound decision making. For example, what is the content that various students should learn in secondary school? What are the most appropriate study materials and methods? Which assessment methods could steer the teaching and learning toward worthy objectives and also accurately reflect the students' level of knowledge? These serious questions and others like them deserve a strict and systematic study, but they are beyond the scope of this document.

This document focuses on math teachers as the school-related factor that has the greatest impact on the students' math education.

This document focuses on math teachers as the school-related factor that has the greatest impact on the students' math education. The first question the committee discussed under the mandate it received was: "What do those engaged in teaching mathematics in secondary schools need to know?" That is, what are the components of knowledge required for the skilled, effective and successful work of math teachers? From the outset, the committee members agreed that the math teacher (like any other teacher) should have a broad education in various fields (including art, literature, history and philosophy) and, in particular, the teacher should be fluent in the language of instruction (Hebrew or Arabic).

But the work of the committee concentrated on defining the specific knowledge for the teacher's professional work in math class. The answers to the question about specific knowledge in mathematics have repercussions for the training of teachers before they begin teaching, for their registration and certification, for their continued training during the course of their professional life, and more. Therefore, the committee decided to address this complex of issues.

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As the committee delved into these questions, it became clear that it was necessary to widen the purview by amending the wording of the question: "What do those engaged in teaching mathematics in secondary schools need to know?" The deletion of the element of knowledge from the wording of the question was not because knowledge is unappreciated. Rather, it stems from the realization that it is essential to define the needs of the teachers (the physical environment, intellectual environment, work conditions, mutual support, and more) and to support them so that the components of knowledge can develop in optimal conductions and reliably serve the teachers in their work. The committee regarded the specification and description of the desired components of knowledge for the teacher as a necessary but insufficient condition for the effective teaching of mathematics.

Consequently, the document summarizing the committee's work begins with this introductory chapter describing the background of the committee's work, the state of math studies in secondary school education in Israel, and the place of the teacher's knowledge among the factors that affect the students' achievements. The third chapter describes the components of knowledge the committee believes are required for those engaged in teaching mathematics in secondary school education, and the document later addresses the question of how to ensure that teachers indeed acquire this required knowledge. The fourth chapter reviews the questions pertaining to the training of math teachers and presents the committee's recommendations on this issue. The fifth chapter deals with the certification exam for teaching mathematics, an exam aimed at ensuring the quality of training. The issue of the continued professional development of math teachers during their years of work is discussed in the sixth chapter. The committee deemed it appropriate to devote the seventh chapter to the training of the teachers of teachers; the former comprise the main factor in the professional development of classroom teachers. The evaluation of teachers is discussed in the framework of the eighth chapter, this time as an element in the professional development of the teachers. Finally, in the ninth chapter, additional recommendations of the committee are presented – recommendations pertaining to the various factors in the teachers' environment that are likely to help fully utilize their knowledge and abilities.

2.2 The state of math studies in secondary school education in Israel

During the past decade, a series of validated and calibrated tests – local and international – were conducted in Israel under the auspices of the National Authority for Measurement and Evaluation (RAMA).

During the past decade, the number of students tested in the matriculation exams in mathematics rose significantly, and the average score of the students also rose. According to figures from the Central Bureau of Statistics (CBS, 2010), the number of students who took the matriculation exam in mathematics at a level of 4 or 5 units increased by about 37% between 1997-2007, and the number of examinees at a level of 3 units in math rose by about 20%. However, it should be noted that relative to the population in this age group, the percentage of those tested at the higher level (4 or 5 units) in mathematics remained almost unchanged (about 23% at 4 units and about 16% at 5 units), while the percentage of examinees at 3 units dropped by 10%, from about 50% in 1995 to about 40% in 2007 (and by about 30% in the Arab sector, from 60% in 1997 to about 32% in 2007), and instead the percentage of students tested at the level of 1 unit increased during these years by over 10% (from about 5% to 16% in the Hebrew education system). It should also be noted that during these years the percentage of schools where students were tested at the higher levels (4 or 5 units) declined by about 10% (from 89% to 80% in the Hebrew state education track and from 90% to 78% in Arab education), with the exception of the state religious education track, where the opposite trend was found (an increase from 81% to 89%). In the overall Hebrew education system, the average matriculation score in mathematics rose from 83 in 1995 to 87 in 2007, while in the Arab education system the average matriculation score in mathematics rose from 73 in 1995 to 81 in 2007, though in 2006-2007 the average score declined a bit relative to the preceding years.

The supervisors of teaching at the Ministry of Education are responsible for composing and grading the matriculation exams, and it is not possible to compare the tests from year to year. But besides these exams, a series of validated and calibrated tests – local and international – were conducted in Israel during the past decade under the auspices of the National Authority for Measurement and Evaluation (RAMA). In these tests, the raw scores were converted to a scale of scores with an average of 500 (in the base year) and a standard deviation of 100.

The Meitzav[3] tests (indexes of school effectiveness and growth) are calibrated tests composed by RAMA according to the Israeli curriculum. They are administered, inter alia, in 8th grade in four core subjects including mathematics. The Meitzav results in mathematics in 8th grade showed continual improvement from 5768 (the 2007-2008 school year) to 5771 (the 2010-2011 school year). The test data indicated a disparity of one full standard deviation between students from a high socio-economic background and students from a low socio-economic background.

[3] From a lecture by Prof. Michal Beller, the director-general of the National Authority for Measurement and Evaluation (RAMA), presented to the committee on August 16, 2010. And from RAMA's website: http://cms.education.gov.il/educationCMS/units/rama (in Hebrew)

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The PISA (Program for International Student Assessment) test is an international exam administered once every three years among a representative sample of 15-year-olds in various countries – in mathematics, language and science. It tests literacy and is not necessarily connected to the curriculum in the various countries. The PISA 2009 results in mathematics indicate that Israel's average score (447) is far below the average of OECD countries (496), placing it in 41st place among the 64 countries participating in the study that year. In addition, the percentage of Israeli students scoring at the highest levels was only 6% (well below the OECD average of 13%), while the Israeli students scoring at the lowest levels stood at 39% (compared to 22% on average in OECD countries). The (lowest) threshold score for the top 5% of Israeli students was below the OECD average for the top 5% of achievers (615 versus 643, respectively). Similarly, the (highest) threshold score for the weakest 5% of students in Israel was below the OECD average for the lowest 5% of achievers (272 versus 343, respectively). Between the years 2006 and 2009, there was a small improvement (5 points) in the scores of Israeli students in math literacy (and a total increase of 14 points since 2002). The percentage of students at the highest levels (5 and 6) did not change, and the percentage of students at the lowest levels decreased by 3%.

TIMSS (Trends in International Mathematics and Science Study) is an international exam that tests the content studied in the science and mathematics curriculum in 8th grade. It is administered once every four years by the International Association for the Evaluation of Educational

Achievement (IEA). In general, the curricular framework of this exam corresponds to the program of studies in Israel in the subjects of mathematics and the sciences. In 2007, the fourth exam was administered in the series of TIMSS research in 49 countries and in 7 educational jurisdictions. The TIMSS research includes weaker countries than those participating in the PISA exams. Therefore, Israel's score ranks it closer to the international average (24th of 49 countries): The average Israeli score was 463, a bit higher than the average of the countries participating in TIMSS in 2007 (451). In Israel, the percentage of students scoring above the "Advanced" threshold (a score of 625 and above) was only 4%, while the percentage of students who scored below the "Low" threshold (400 or less) was 25%. The scores of the Israeli students were quite similar in the various fields of mathematics: numbers (469), algebra (470), data and probability (465). The exception was the field of geometry, in which the average score (436) was low relative to the average of the participating countries. In mathematics, the students in Israel achieved higher scores on "application" questions: 473 versus 456 on "knowledge" questions and 462 on "thinking" questions. The average score of 8th graders in Israel in mathematics in 2007 was similar to that of 1999 (463 and 466 points, respectively). In 2003, a higher average score was recorded than in these two years (496 points).

Data from the Central Bureau of Statistics also indicates that during the years 1991-2005, there was a continual decline in the psychometric scores of matriculation examinees in mathematics at all levels of testing, together with a decrease in the percentage of students taking the psychometric test. However, the overall and quantitative scores on the psychometric test of new undergraduate students at universities rose between the years 1995-2010. The psychometric scores of math students rose during these years to a lesser extent than the average psychometric score of the rest of the students. Thus, in effect, the achievement level of math students in the universities declined.

The problem of the math achievements of Israeli students relative to various countries in the world can also be seen in the context of the concern felt in many countries about the low level of interest by young people in the STEM (Science, Technology, Engineering and Mathematics) subjects, which are considered vital for a modern economy. (See for example: Osborn, 2008; Kuenzi 2008).

2.3 Factors affecting students' achievement level in mathematics

A student's learning and achievement are influenced by many factors, such as the student's aptitude, goals, conduct and motivation; the family's resources, its views and the extent of support it provides; fellow students, their aptitude, views and conduct; the way the school is organized, its resources and the school climate; the content and structure of the curriculum; and the teachers' skills, knowledge, views and methods of instruction. The impact of the various factors on learning and the connections between these factors are at the heart of educational research, but the great diversity among students and between educational systems makes it difficult to reach conclusions based on samples (OECD, 2005).

In recent decades, large databases have been compiled in the United States that have enabled a systematic study of the influence of various factors on students' achievements.[4] These efforts have included collecting a substantial amount of data on students, teachers, principals, schools, curricula and the connections between them. The data was also collected consistently, over the years, thus making it possible to analyze the factors of change and their long-term impact. (See for example: Darling-Hammond, 1999.)

Among the factors that are more directly affected by educational policy, the centrality of the teacher is consistently noted as influencing the students' achievements.

A salient and consistent finding in research of this type is that the disparity between students' achievements is largely attributable to factors that lie outside of the direct purview of the education system, such as family and social background, as well as aptitude and views (OECD, 2005). Nonetheless, among the factors that are more directly affected

[4] For example, the databases: Schools and Staffing Surveys (SASS), National Assessment of Educational Progress (NAEP)

by educational policy, the centrality of the teacher is consistently noted as influencing the students' achievements, as opposed to other factors such as the school, the class size and the curriculum. Research conducted on data from the state of Tennessee in the United States found that the cumulative impact of teachers can widen the gap between similar groups of students by up to 50% within three years (Sanders & Rivers, 1996). In a 2007 McKinsey report (McKinsey, 2007), this finding was described in the following sentence: "The quality of an education system cannot exceed the quality of its teachers."

However, it is not simple to define the characteristics of a good teacher: It was found that various administrative parameters, such as education and experience, are not always connected to the students' achievements, or there is a weaker correlation than expected. (For an extensive review of the literature on this subject, see Goe, 2007). However, among secondary school math teachers, who are the focus of this document, a connection was indeed found between the teachers' knowledge and the students' achievements, as explained below in this document – and this was the focus of the committee.

Among secondary school math teachers, who are the focus of this document, a connection was indeed found between the teachers' knowledge and the students' achievements.

> The committee was asked to address the knowledge required for those engaged in teaching mathematics, while recognizing the relative importance of this factor among the characteristics of a good teacher. Of course, this does not detract from the importance of the other qualities of a good teacher, such as leadership and the ability to motivate, enthusiasm and creativity, values, interpersonal relations and various characteristics.

Chapter 3
Components of Knowledge for Teaching Mathematics

This chapter describes the components of knowledge required for math teachers in the secondary schools. These components will be discussed according to the following categories:

- ★ Content knowledge (math knowledge)
- ★ Pedagogical knowledge
- ★ Practical knowledge

Based on these components of knowledge, recommendations will be made regarding guidelines for training programs.

3.1 Mathematical content knowledge

It is clear to everyone that math teachers must know the subject matter they are teaching. But this statement does not define with sufficient precision the types of mathematical knowledge required for teaching, or the depth and breadth of this knowledge. Therefore, the following division of content knowledge is proposed:

> - ★ General mathematical content knowledge, as studied in the framework of undergraduate studies in mathematics
> - ★ Mathematical content knowledge related to the content of math instruction in secondary schools
> - ★ Mathematical content knowledge as expressed in various fields
>
> The three elements of content knowledge are interwoven, but each has its own features and purpose.

3.1.1 General mathematical content knowledge

This includes knowledge studied in the framework of an undergraduate degree in mathematics (infinitesimal calculus, linear algebra, abstract algebra, probability, and more). This knowledge provides depth, breadth and an overview for the content that teachers introduce in the classroom, and offers tools for the analysis and synthesis that are at the heart of all mathematical work. For example, it develops awareness of the need for definitions and basic assumptions, the need to prove or refute assertions, and role of examples and counter examples. This knowledge also constitutes a basis for the ability to move from an intuitive idea to its formal expression. Content knowledge also establishes mathematics as a language, as a body of knowledge that is built layer by layer according to the rules of logical deduction, and as a tool for the uncompromising search for scientific proof. In addition, in-depth knowledge in the field of content is an essential component in the self-confidence of teachers, in their ability to answer unexpected questions from students, and in their ability to provide a "mathematical horizon."

3.1.2 Mathematical content knowledge related to the content of math instruction in secondary schools includes:

A) Observing the math content studied in schools from the perspective of a professional mathematician, in the spirit of Felix Klein's approach. Klein surveyed the fields of high school arithmetic, algebra, analysis and geometry with advanced mathematical tools (Klein, 1908[5]) with the aim of offering teachers a mature look at the mathematics they themselves learned in school and "to accommodate" this content in the world of the contemporary mathematics of his time.[6]

[5] The original books of Felix Klein were published in Germany and can be found in their original version at the following site: http://www.archive.org/stream/elementarmathem00kleigoog#page/n4/mode/2up. The first edition of the books in English translation was published in 1939 and many subsequent editions have been published. For example: *Elementary Mathematics from an Advanced Standpoint: Geometry*, Dover Publications, 2004 and *Elementary Mathematics from an Advanced Standpoint: Arithmetic, Algebra, Analysis*, Dover Publications, 2009.

[6] One hundred years after the appearance of Klein's books, the importance of this information received renewed attention from the International Commission for Mathematical Instruction (ICMI) and the International Mathematics Union (IMU). In a joint international effort initiated by these organizations, "The Klein Project – 100 Years Later" was launched with the aim of

B) Knowledge of mathematics that is accessible to students but is not included in the program of studies. There are many subjects at the level of secondary school mathematics (subjects that can be learned on the basis of the students' prior knowledge and abilities) that are not covered in the classroom. This can include entire fields (for example, number theory or discrete mathematics in general) or properties, theorems and problems in algebra, geometry and analysis that are defined as areas of "enrichment." That is, they do not require advanced mathematical knowledge and do not constitute an essential foundation for continued studies. This knowledge expands the set of practical tools available to the teachers, who can use this to add variety to their teaching and as regular instruction in classes that learn at a quicker pace.

A synthesis of elementary knowledge and advanced knowledge provides concrete tools for the teacher to enrich the world of the students, whether in the form of planned instruction (enrichment subjects) or as a response to unanticipated mathematical incidents in the classroom.

3.1.3 Mathematical content knowledge as expressed in various fields, including:

A) Mathematical applications in other fields of knowledge, such as engineering, computer science, economics, architecture and art, and particularly in fields studied in school, such as chemistry, physics and biology. A certain level of knowledge in these applications will enable teachers to make connections between them and the mathematics they are teaching. There is special importance in emphasizing that mathematics is the language of science and in noting its centrality in the development of modern science.

B) "Mathematical literacy": This term refers to the ability to connect mathematics with its applications and expressions in the everyday life of every person – the ability to critically read data in various representations,

adapting Klein's approach to the mathematical knowledge that developed and accumulated during the course of the 20th century. For details, see: http://www.didaktik.mathematik.uni-wuerzburg.de/projekt/klein/project.html the ability to rely on mathematical tools in making sound decisions, and so on. During the past decade, this knowledge has received special attention in many countries, and underlies the development of the PISA international exam.[7] In light of the centrality of "mathematical literacy" in our era, we will elaborate on this subject below.

There is an additional type of specific mathematical knowledge for teaching that is expressed almost only in the work of the teacher, but because of the connection between this type of knowledge and pedagogical knowledge, we will discuss it in the next section.

3.2 Pedagogical knowledge

Pedagogical knowledge pertains to the unique work of each teacher in each field of content. This knowledge can be divided into two components:

- ★ General pedagogical knowledge – including familiarity with cognitive, social, emotional and cultural aspects in the world of the students, familiarity with various teaching approaches, and so on.
- ★ Pedagogical knowledge that is unique to the field of content (in this case, mathematics) – including knowledge about the students' ways of thinking, typical difficulties students experience when learning particular mathematical topics and ways of contending with these difficulties, knowledge about the choice of a teaching strategy for a particular topic, and so on. This type of knowledge includes what the professional literature calls "pedagogical content knowledge," a term coined by Lee Shulman.[8]

Below is a description of types of pedagogical content knowledge that is unique to teachers of mathematics, as defined by members of the committee.[9]

[7] See, for example, http://www.oecd.org/dataoecd/46/14/33694881.pdf
[8] For example, see the classic article on this subject: Shulman, L. S. (1986).
[9] It should be noted that the committee deemed it appropriate to categorize the types of knowledge in a slightly different way than usually appears in the literature.

Mathematical knowledge includes a mathematical ability to assess the correctness and partial correctness of assertions and to analyze students' explanations, assumptions and various methods of problems-solving, an ability to analyze the mathematical ideas that are (or are not) expressed in a particular task, knowledge of various representations of concepts in mathematics and an ability to incorporate them in teaching, and so on.

3.2.1 Knowledge of mathematics for teaching:

Each field of content makes use of aspects of mathematics that are unique and essential to it. For example, the mathematical content required for economics is not necessarily the same content and areas of emphasis required for physics. This fact led to the recognition that there is mathematical knowledge unique to teaching.[10] This mathematical knowledge includes a mathematical ability to assess the correctness and partial correctness of assertions and to analyze students' explanations, assumptions and various methods of problems-solving, an ability to analyze the mathematical ideas that are (or are not) expressed in a particular task, knowledge of various representations of concepts in mathematics and an ability to incorporate them in teaching, and so on.

3.2.2 Didactical mathematical knowledge:

This field of knowledge focuses on such questions as: Which teaching methods are appropriate for the specific content? How is it possible to organize the content in a way that will be understood and easier to learn? What makes certain concepts difficult

[10] Deborah Ball developed the concept and coined the term "mathematical knowledge for teaching." See, for example: http://sitemaker.umich.edu/lmt/people. Though Ball primarily refers to mathematical knowledge for elementary school, there are studies based on this concept in the context of mathematics for secondary school. For example, see: http://horizonsaftermath.blogspot.com/2011/09/if-you-think-you-know-it-try-to-teach.html. Ball presented her research on the subject of "Mathematics for Teaching" in a seminar conducted by the committee at the Israel Academy of Sciences in Jerusalem (January 2011). A summary of Ball's method and a summary of the discussion about it appears on the Internet site of the Initiative for Applied Education Research: http://education.academy.ac.il/hebrew/PublicationDetails.aspx?PublicationID=25&AreaID=&QuestionID=-1&FromHomepage=True (Hebrew)

or easy to understand? Which examples or non-examples should be chosen to minimize the risk of creating an erroneous image of the concept? What aspects of the things the students bring to the learning environment can accelerate or delay their advancement? What range of teaching measures is available? How should the teacher approach special populations (struggling students, outstanding students, or students from a different cultural background)?

3.2.3 Curricular knowledge:

This field of knowledge entails familiarity with various programs of study in mathematics (syllabuses, textbooks, guidebooks for the teacher, evaluation kits), familiarity with their objectives, and the ability to analyze and identify their advantages and disadvantages. This component includes the knowledge upon which the teachers develop resourcefulness in finding curricular resources (books, magazines, technological accessories, Internet sites) and adapting them to their teaching.

3.2.4 Knowledge about evaluating learning and achievement

This component of knowledge includes a teacher's ability to design standard and alternative evaluation tools or to use them to monitor the progress of the students' learning. It also includes familiarity with national and international evaluation policy, an ability to critically relate to test items and an ability to determine what can or cannot be deduced from the results of employing various evaluation tools.

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3.3. Practical knowledge

This knowledge directly refers to the everyday work of math teachers in their classrooms. This field of knowledge also has a general component (lesson management, contending with discipline problems, conduct within the school system, etc.) and a component that is unique to mathematics. The unique component is based on a practical translation of all of the elements described in the previous sections and the integration of them. It includes composing and implementing a lesson plan, and the ability to reflect on a lesson and teaching practices, redesigning them as needed, and so on.

> The committee recommends that programs for teacher training and programs for the professional development of teachers be based on developing, consolidating, enriching and deepening all of the components of knowledge described above. In the following chapters, a framework for programs of study will be proposed, along with principles and recommendations for its implementation.

Due to the uniqueness of two fields – the history of mathematics and math literacy – and because of the possibilities they offer for developing and integrating different types of knowledge, they are addressed in detail below:

The history of mathematics as a component in the teachers' mathematical background

There is a consensus among math educators today that the history of mathematics is a field that offers potential for developing the various types of knowledge. For example:

★ **Strengthening mathematical knowledge:**
History enables the enrichment and deepening of all of the subtypes of general mathematical knowledge described in section 3.1.1. For example, history constitutes a rich source of mathematical problems that derive from engaging in pure mathematics or that arise from problems in other fields. History also notes the development of formal mathematical knowledge, the reasons for its development, and more.

★ **Strengthening unique mathematical knowledge for teaching:**
The frequent use of representations, symbols and ways of thinking make them self-evident for those who know mathematics, including teachers. The automatic nature of this use blurs the underlying principles and insights, which are so vital for the work of teaching. The exposure to historical development enables one to breach the boundaries of convention and thus reassess both the complexity and power of the

symbols, forms of representation and ways of thinking, and to reawaken awareness of the fact that they did not "descend from the heavens" but are the product of human creation over many years.

Another aspect of strengthening mathematical knowledge for teaching pertains to a teacher's ability to listen to what the students are saying, to try to decipher the underlying mathematical principles and to consider all this when developing the teaching process. This task is complex and requires ongoing study and practice, and the history of mathematics can play a role in this. When teachers contend with an historical text that addresses concepts, qualities or solutions whose content is familiar to them, but are unfamiliar with ways of dealing with them (other methods, various representations and symbols, non-standard considerations, different language, and so on) – a situation is created that is similar to classroom situations in which the teacher is required to listen to something unexpected. When dealing with an historical text, it is more difficult to reject a method or idea as incorrect. An effort is required to "get into the head" of the author of the text in order to try to understand the uniqueness of addressing the subject in an unanticipated way. Repeated exercise of these types of situations is likely to develop the desired ability to listen or to improve its flexibility.[11]

★ **Strengthening pedagogical-didactic knowledge:**
In general, mathematics is represented (by both mathematicians and teachers) in a polished, sometimes detached way, disconnected from time and place, super-human and, as such, leaving no room for doubt. It is rare that students and teachers encounter mathematics during its creation or development, processes that include assumptions and doubts, involving intuitions, mistakes and even experiences of entering a dead end and a failure to understand the connection between close topics. The history of mathematics, and particularly the encounter with original sources, provides a peek into mathematics as it evolves, while giving explicit expression to conceptual processes and attempts to resolve misunderstandings or contradictions. An additional aspect

[11] For experimental work in this context, see for example: Arcavi and Isoda, 2007.

of strengthening the pedagogical-didactic knowledge that can find expression in history is the posing of fundamental didactical issues such as the use or non-use of metaphors and non-mathematical illustrations, with their advantages and disadvantages.

★ **Strengthening the curricular and practical knowledge:**
The history of mathematics enables the study of math education – for example, the existence, development, success or failure of various curricular approaches in different periods of time and in different places.

> In light of all of the aforementioned, we recommend incorporating the history of mathematics in all frameworks of training math teachers (new teachers, as well as experienced teachers engaged in ongoing professional development), and we recommend devoting attention to ways of doing this.

Thus, for example:

★ The study of history should focus not only on facts (people, their work, dates, chronologies) and anecdotes, but primarily on the conceptual development of the topics. For this purpose, it might not be necessary to survey various aspects of one time period after another, but instead study the development of a selected topic over the course of time.

★ The learning should be active and experiential, and should included contending with and deciphering original texts, as well as explicit connections to mathematical problems and to central pedagogical and cognitive problems (such as the inherent difficulties in a particular subject).

★ For the purpose of training teachers, it is desirable to develop unique educational materials on the history of mathematics based on past attempts to develop similar materials – all this according to the guiding principle that the history of mathematics can be an important tool for developing and deepening all of the components of knowledge for teaching mathematics that are described in this chapter.

★ There are different models for developing courses on the history of mathematics: There are courses that cover the entire history (from the time human beings began to engage in arithmetic and until today); some teach particular mathematical problems according to historical development; and there are many others. The committee is not recommending a particular model, but suggests that each model adapt these principles as a basis for developing the course.

★ The study of history must beware of the dangers the literature warns about (Fried 2001, for example) and, in particular, the tendency to draw conclusions about various eras in the past based on contemporary knowledge.

'Mathematical literacy' as a component in the math background of teachers

Formal mathematical knowledge, as a highly valued cultural asset and also due to its practical importance, has always been considered an essential part of school education. In the past decades, a type of knowledge called "mathematical literacy" has been defined, and it is gradually making its way into education systems in the world and into some of the international exams designed to assess the students' knowledge.

The term mathematical literacy refers to the ability to employ mathematical thinking outside of an immediate mathematical context. This includes the ability to use mathematical considerations and processes in carrying out everyday tasks and in decision-making processes. Literacy knowledge, therefore, has two main and interrelated components: the ability to think in mathematical ways (knowing the "how") and skill in identifying contexts and situations in which this type of thinking is liable to produce a special benefit (knowing the "when"). It should already be noted that the need to equip teachers with the knowledge that will enable them to develop their students' mathematical literacy poses a special challenge for the teachers themselves and for those who train them. This is because of the phenomenon known as the "situatedness" of learning (the dependence of the developing knowledge on the context in which it is learned, a dependence that leads to its lack of availability in other contexts) and because of the paucity of studies that address the question of how it is possible to overcome this "situatedness."

There are various definitions of mathematical literacy. For example, it is said that mathematical literacy is designed to provide each student with "a broad understanding of mathematical concepts and the ability to know when and how to use them" as well as "analytic skills and the ability to deduce conclusions" (QCA 2005, p. 2). It is also said that mathematical literacy will equip the learners with "an awareness of the role of mathematics in the modern world and with an understanding of this role" (Department of Education (DoE), South Africa, 2003, p. 9). The committee recommends adopting the following definition (Gal 2012, p. 3): "a combination of foundations of knowledge, skills, views and motivational elements required for effectively contending with various life-tasks that have a mathematical or statistical component. According to this approach, the salient elements of mathematical literacy include quantitative and statistical literacy, financial literacy, mathematical literacy in the medical-health context, and mathematical components in science literacy. Thus, "five main bases of knowledge and cognitive abilities should be fostered (mathematical knowledge, statistical knowledge, linguistic knowledge, knowledge about the context or the world, a list of critical questions) and a number of views, beliefs and motivational components (such as a perception of one's own capability in the field of mathematics, a critical outlook, etc.) that a mature person needs in order to effectively contend with various life-tasks that have a mathematical or statistical component" (Gal 2012, p. 43).

The main characteristic that differentiates mathematical literacy from mathematical knowledge is the emphasis on the applications of mathematics, and particular those accessible to each educated person. One of the prevalent reasons for including formal mathematics in school curricula is that this knowledge comprises a collection of universal tools used in many life-tasks. Proponents of mathematical literacy demand more than that: They do not suffice with developing and becoming familiar with the tools; they also demand an assessment of the need and development of the ability to apply them. Mathematical literacy turns the person into an active and skilled user of mathematical and statistical knowledge in any situation in which this is likely to yield benefit. Due to the emphasis on applicability, the mathematical discourse that develops as part of a literacy approach is not necessarily identical in form and content to that of traditional studies.

Teachers are required to have good knowledge of the language of instruction (Hebrew or Arabic). Maintaining precision and tidiness of language is recommended in every field and in every context, but mathematics requires special meticulousness in language.

In addition to all this, teachers are required to have good knowledge of the language of instruction (Hebrew or Arabic). Maintaining precision and tidiness of language is recommended in every field and in every context, but mathematics requires special meticulousness in language. Unlike tangible objects that are familiar to the students from everyday life, the objects studied in math class are presented to the student primarily through examples and linguistic definitions. Therefore, imprecision in mathematical wording is liable to distort the mathematical world of the student. Mathematics has explicit and formal rules that protect it from such distortions and make the mathematical discourse immune to internal contradictions. Learning to strictly follow these rules is an integral part of studying mathematics, and a byproduct of this meticulousness is a deeper understanding of the process of language and a greater appreciation for the proper use of language beyond mathematics too.

> The committee deems it appropriate to include mathematical literacy, in all its aspects, among the components of knowledge required for teaching mathematics: the meaning and content of this literacy, the different approaches to it, the way it is expressed in various education systems in the world and in international exams, its connection with formal mathematics, and more.

Familiarity with mathematics as the language of science – a component of the knowledge of math teachers

It is important to highlight the special connection between the mathematics studied in high school and the rest of the natural sciences, and physics in particular.

Together with the emphasis on literacy and the importance of the history of mathematics, it is important to highlight the special connection between the mathematics studied in high school (functions, differential calculus, probability theory) and the rest of the natural sciences, and physics in particular. Indeed, the scientific revolution of Galileo, one of whose central components was a description of natural phenomena in the language of mathematics, led to far-reaching changes in human thought. This development provides a golden opportunity to underline the full significance of mathematics for understanding the world around us, and the committee recommends that the teachers' knowledge include familiarity with this historical process and with the important developments that followed in its wake. A special effort should be made to define the type of connection between the mathematics the students are learning and the significant applications that stemmed from the scientific revolution.

Chapter 4
Teacher Training

4.1 Introduction

In the previous chapter, we surveyed the various areas of knowledge that teachers should acquire during the course of their training. But it is also important to address the practical aspects. In particular, it is important to address questions such as: How much of the training should be devoted to each area? What should the duration of the training be? What model of training should be employed? What should be the criterion for satisfactory training and how should it be monitored?

In this chapter, these questions will be discussed, with a comparative look at various countries.

Recommendations will be presented at the end of the chapter.

4.2 Duration of training and content

4.2.1 Duration and content of training in various countries

A) Duration of training

In European countries, the duration of training programs for teachers in middle school is 4.4 years on average, and training programs for teachers in high school last 4.9 years on average. In most of the countries, the duration of training is 4 years, but in some countries there are longer programs lasting up to 8 years, as in Italy for example. In Bulgaria, the training for middle school and high school is 4 years, and the training is conducted

In most of the countries, the duration of training is 4 years, but in some countries there are longer programs lasting up to 8 years.

in the framework of the mathematics department in universities. On the other hand, Germany trains teachers in a separate institution for 1.5 to 2 years after they complete their discipline-specific studies in university at a bachelor's degree (B.Sc.) level, and the training continues in some cases for 6.5 years. In Finland, teachers complete a master's degree in the framework of their teacher training. (OECD, 2005; Schmidt et al., 2007).

While the duration of training varies among the countries of Europe, and even within these countries, there are many countries in the world where 4 years of training is customary for teaching at all grade levels. In England, Australia, Canada (Quebec) and Mexico the duration of teacher training is 4 years. In Mexico, the training is conducted in the schools themselves, and not in institutions of higher learning (Babcock et al., 2010). In the United States, the program of training for math teachers in middle schools is also 4 years (and 5 years in a few institutions) and the training conducted in the framework of the school or in university education departments in three different tracks: certification to teach math in grades 1 to 8, certification to teach math in grades 6 to 8 or 7 to 9, and certification to teach math from the middle school and above – through grade 12 (Babcock et al., 2010). In some of the states in the U.S. the teachers are required to complete a graduate degree within several years of entering the teaching profession. In most cases, the teachers pursue a graduate degree in education and not in a particular discipline. Research has not found evidence that this graduate degree improves the teacher's ability (Rivkin et al., 2011 in OECD, 2005), and some people have expressed concern that this requirement actually deters potential candidates for teaching (Murnane 1996, in OECD, 2005).

In Taiwan and South Korea, the duration of teacher training for middle schools and high schools is 4 years, and the training is conducted in special universities that are certified for training teachers (Schmidt et al., 2007). In Shanghai, the educational requirements for teachers have been gradually raised in recent decades. Today, secondary school teachers are required to hold a bachelor's degree in the profession, and many of them have a graduate degree (OECD, 2011).

Trends: In many countries, the training for teachers has already been lengthened in recent decades, and is expected to become longer. The Bologna

Accords of 1999 mandate a gradual process of reducing the disparity in the higher education systems in the countries of the European Union (EU). In the field of teacher training, the trend is for the training to be conducted in a university institution, according to the serial model (OECD, 2005)[12]. A recent EU report surveys the training programs for math teachers in Europe and describes the trend in many EU states to lengthen the training of math teachers. In Ireland, for example, the Ministry of Education encourages teachers to study for a unique master's degree developed in collaboration between the ministry and the universities, and the plan is to increase the duration of teacher training to 2 years (in the serial model) for those who have undergraduate degrees. In the past, the training of secondary school teachers in Spain included participation in 150 hours of pedagogical courses after completing a 4-year undergraduate degree (B.Ed.), but the pedagogical courses were recently replaced by one year of academic studies toward a graduate degree. Since 2011, teachers in Iceland are required to complete a master's degree as part of their training for teaching (Eurydice, 2011).

On the other hand, the shortage of teachers and candidates for teaching in many countries (exceptions include Taiwan, Japan, Ireland and Belgium) has led the countries to loosen their requirements and offer training paths adapted to this real shortage (Morris & Williamson, 2000 in: Cooper & Alvarado, 2006). In most of the countries of Europe (Denmark, Holland and Sweden, for example), graduates who did not study education are offered training programs with greater flexibility in the number of hours of study and which enable teacher training in parallel to work or via distance learning (OECD, 2005). In the United States, England, Holland, Wales (and in most of the countries studied), due to the growing shortage of teachers and the low social status of the teaching profession, trainees are starting to teach just a few weeks after beginning their training.

Shortened ("alternative") training programs in the United States have succeeded in attracting quality candidates who otherwise would not have engaged in teaching, and in some states they comprise as many as 25% of new teachers (Darling-Hammond, 1999). In a study conducted in the United

[12] See chapter 4.4 below regarding the serial model and the parallel model.

States, no difference was found between teachers who were trained in the standard programs and those trained in alternative programs, and the disparity within each group was much larger than the difference between the groups (Ballo & Podgursky, 1999; Raymond et al., 2001, in OECD, 2005). Other studies seeking to investigate this question focused on different programs and different parameters, making it impossible to compare their contradictory findings (Boyd, Grossman, Lankford, Loeb, and Wykoff, 2006, in: National Mathematics Advisory Panel, 2008). Due to the doubt the studies have cast upon the effectiveness of prolonged training, the OECD organization has suggested reconsidering the benefit of prolonged training, and has recommended weighing in-service professional development as an alternative[13] (OECD, 2005 p. 105).

Among high school math teachers a positive correlation was consistently found between the number of math courses the teachers took and the number of their math degrees – and the achievements of their students.

A comprehensive review of the research literature written during the years 1975-2007 on the connection between the level of teachers' math knowledge and the achievements of their students, found that the research studies can be categorized according to the various indexes they used to measure the teachers' knowledge – the type of certification, the courses the teachers studied, the degrees they completed, and the direct tests of knowledge they took. Studies that used certification as an index of knowledge did not yield unequivocal findings, so it cannot be said that a connection was found between certification and students' achievements. Research that examined the math knowledge of elementary school and middle school math teachers via indexes of the courses they studied or via the degrees they completed – also failed to yield consistent findings. However, among high school math teachers a positive correlation was consistently found between the number of math courses the teachers studied and the achievements of their students. Similarly, a positive correlation was found between the number of math degrees the teachers completed and their students' achievements. Studies that made use

[13] The topic of professional development will be discussed below, in chapter 6.

of direct tests of the teachers' knowledge also indicated, in general, a positive correlation between the teachers' achievements on the tests of knowledge and the achievements of their students (National Mathematics Advisory Panel's task group, 2008).

In analyzing the connection between data on math teachers' education and the achievements of students on matriculation tests in mathematics (and the level at which they were tested),[14] a positive connection was found between the percentage of math teachers in the school who have a relevant educational background[15] or have a master's degree, on the one hand, and the students' level of math study (the average units of study in mathematics and the percentage of students tested at the higher level) and their achievements on the matriculation exam in mathematics, on the other hand. Similarly, a positive connection was found between the percentage of math teachers in the school who are certified to teach mathematics in high school[16] and the level at which the students are tested in the math matriculation exam (Central Bureau of Statistics, 2010). However, it is important to note that his connection is not necessarily causal.

B) Content of the training

The training programs for teachers generally include courses of discipline-specific content, methodological courses, education and psychology courses (for example, developmental psychology and the history and philosophy of education), as well as practice teaching. However, the programs differ in emphasis and in how they divide time between the various components of the training. In some of the countries, an effort was made to equip future teachers with the ability to conduct independent research (OECD, 2005).

In terms of the time devoted to practice teaching, there is great disparity between the training programs in Europe – ranging from a single course

[14] The data refers to the students' achievements; improvement in these achievements was not studied.
[15] The data was examined primarily among young teachers whose educational data could be accessed.
[16] Certification data was also available for teachers over the age of 50, and provides an indirect indication of a relevant academic degree and teaching certificate in mathematics.

at the end of the training to an entire year. But the trend is to incorporate practice throughout the training, particularly in the integrative track, in order to minimize the problems often associated with practical work – such as a disconnection between practice and theory, experience in only a small part of the roles of the teacher, a shortage of mentors, gaps and lack of coordination between mentors in the school and faculty in academic training (OECD, 2005). New and experienced teachers have emphasized the importance of practice teaching during their period of training (Wilson et al. 2001, in: OECD, 2005), and research has found that the percentage of teachers who continue in the profession is higher among teachers who did more practice teaching during the course of their training (Fleener, 1998, in: OECD, 2005). Nonetheless, studies show that practice teaching in itself does not ensure better training for teaching mathematics, and one of the central problems researchers and teacher-trainers face in many countries is how to construct the component of practical work in a way that contributes to the training of math teachers (Even & Ball, 2009).

In a comparative study conducted in 2007 in a variety of training institutes in six countries (Taiwan, South Korea, Bulgaria, Germany, Mexico and the United States) (MT21- Mathematics Teaching in the 21st century), the level of mathematical and pedagogical knowledge of math teachers in the middle schools was examined at the end of their training. Significant differences were found between the various countries: The achievements in Taiwan and in South Korea were higher in all of the subjects; the achievements in Germany were average and in Mexico they were the lowest. The achievements of teachers in the United States were particularly low in the fields of algebra, functions and geometry, and average in the field of statistics. In comparing the various training programs within the U.S., it was found that the achievements of graduates of training programs for teaching mathematics in secondary schools (grades 7 to 12) were higher by almost one standard deviation than those of their colleagues who graduated from training programs for elementary education (grades 1 to 8) or middle school (grades 6 to 8). The differences in the level of the teachers' knowledge were found to correspond to the differences in the amount of time the teachers said they devoted to the listed subjects of study during their training. In Taiwan, between 84% and 95% of the math subjects and about 70% of the general and mathematical pedagogical subjects were studied, while in Germany the students learned mathematics at a moderate scope (between 45% to 70%) and only 24% to 30% of the general and mathematical pedagogical subjects. In Mexico, teachers reported an extensive focus on education (70% to 90% of the subjects), and a relatively limited engagement in mathematics (25% to 40% of the subjects). In the U.S., only 42% to 60% of the basic subjects in mathematics were studied on average, and 50% to 60% of the mathematical and general pedagogical subjects. The programs of teacher training in secondary education in the United States included a higher number of advanced courses in mathematics, and less educational knowledge and experience in comparison to the training programs for teaching in middle schools and elementary schools.

The results of the TIMSS exams indicate that the countries whose students received good scores on the exams are the countries whose programs of study set high math requirements and where the future teachers show a high level of knowledge in the field of mathematics. Nonetheless, it should be noted that the low scores of Germany on the TIMSS, despite the relatively high scope of math studies in teacher training, indicates that the math component in training is not the be-all and end-all, and that the other components of training are also valuable (Schmidt et al., 2007).

The results of the TIMSS exams indicate that the countries whose students received good scores on the exams are the countries in which the future teachers show a high level of knowledge in the field of mathematics.

A similar study conducted in 2010 (TEDS-M – Teacher Education and Development Study in Mathematics) examined the level of pedagogical and mathematical knowledge of math teachers in middle schools. The exam was conducted toward the end of their teacher training in a variety of institutions in 16 countries, including the United States. The results of the study indicate that the leading countries are Taiwan, Russia and Singapore. The knowledge level of math teachers in the U.S. is average – but much lower than that of teachers in the outstanding countries. As part of the study, the teacher training programs in the various countries were also examined and significant differences were found in the allocation of time between the training components – mathematics, teaching mathematics, and general pedagogy.

In countries with the highest scores, about 90% of the teachers studied basic courses in mathematics, linear algebra and a yearlong course in infinitesimal calculus, for example.

The two countries in which the middle school teachers received the best scores – Taiwan and Russia – devoted nearly 50% of the courses studied during the training to mathematical content itself, about 30% to the teaching of mathematics and only 20% to general pedagogy. On the other hand, in the United States only 40% of the courses were devoted to mathematics, 30% to the teaching of mathematics and 30% to general pedagogy. In addition, significant differences were found between the U.S. and the countries ranked above it in the percentage of teachers who studied basic courses in mathematics, linear algebra and a yearlong course in infinitesimal calculus, for example. In countries with the highest scores, about 90% of the teachers studied these courses, while in the United States only 67% of the teachers studied a course in linear algebra and just 50% participated in a yearlong course in infinitesimal calculus. Similarly, it was found that in the six countries with the highest scores, the teachers studied a higher number of advanced courses – two more than in the United States (Babcock et al., 2010).

Even so, it should be emphasized that despite the consensus that a strong foundation of knowledge is important, there is no consensus that a broader base of knowledge is better. What is required is sufficient knowledge in a field in order to teach it properly (Cooper and Alvarado, 2006).

> **In summary**, training programs for teaching math in secondary schools in the world usually include discipline-specific courses, general and discipline-specific pedagogical courses, and practice teaching. They are generally four-year programs, and in a few countries the training includes a master's degree requirement. However, the widespread shortage of teachers mandates flexibility in programs and alternative tracks. In the study, no clear advantage was

> found for any particular program of training,[17] but it was found that teachers whose students were high achievers took a slightly higher number of mathematics courses and, in particular, linear algebra and infinitesimal calculus.

4.2.2 Duration and content of training in Israel

A) Teacher training in Israel – Education Ministry requirements

According to the directives of the Ministry of Education,[18] math teachers in grades 7 to 10 are required to have an undergraduate degree in mathematics and an appropriate teaching certificate. Math teachers in grades 11 and 12 are required to have a master's degree (or higher) in the field and an appropriate teaching certificate.

According to the directives of the Ministry of Education, math teachers in grades 7 to 10 are required to have an undergraduate degree in mathematics and an appropriate teaching certificate. Math teachers in grades 11 and 12 are required to have a master's degree (or higher) in the field and an appropriate teaching certificate.

The conditions for receiving a permanent teaching license for grades 7 to 12 are:

> The license will be given to those holding a master's or doctoral degree in the subject for which they seek the teaching license, provided that the degree was awarded by an institution of higher learning in Israel that was recognized by the Council for Higher Education, and provided that the applicant also holds a teaching certificate in this subject that was issued by a school of education in an institution of higher

[17] It should be noted that in all of the examples of successful countries (for example, Taiwan, South Korea and Finland), top candidates were chosen for teacher training (Center for Research in Mathematics and Science Education, 2007, 2010; OECD, 2011). In Finland, for example, it is very competitive to be accepted for teacher training, and only 15% of applicants are accepted. The selection is based on school grades, a test of knowledge, participation in teaching situations, and a personal interview (Kansanen, 2003).

[18] From the teachers' service regulations (2-3 articles 1.21.2), the Ministry of Education's website (Hebrew): http://cms.education.gov.il/EducationCMS/Units/Sherut/Takanon/perek1A/Kabala/Iyuni/kavua.htm

> learning or by an institution recognized for teacher training in Israel, which certifies teachers for elementary school or middle school.

The conditions for receiving a permanent teaching license for grades 7 to 10 are:

> The license will be given to those holding an undergraduate degree in the subject for which they seek the teaching license, provided that the degree was awarded by an institution of higher learning in Israel that was recognized by the Council for Higher Education, and provided that the applicant also holds a teaching certificate in this subject that was issued by a school of education in an institution of higher learning or by an institution recognized for teacher training in Israel, which certifies teachers for middle school or elementary school

There are two main paths in Israel for acquiring an "undergraduate degree in the subject" and a teaching certificate. There are also three possible paths for receiving "a master's degree in the subject" that does not include a teaching certificate.

The Ministry of Education's directives requiring teachers to have an undergraduate (or graduate) degree in the "subject" is not explicit and is open to interpretation. As a result, there is a variety of training tracks possible under the ministry's requirements and, consequently, there is great diversity among the teachers. There are two main paths in Israel for acquiring an "undergraduate degree in the subject" and a teaching certificate. The accepted path in the universities includes a three-year program in the department of mathematics that awards a B.Sc., in mathematics, followed by a two-year program (with one year overlapping) in the School of Education, which awards a teaching certificate in the subject. The accepted path in the academic colleges for teacher training includes a four-year program of study that awards a B.Ed. degree with a specialization in mathematics for secondary schools and a teaching certificate in the subject.

There are also three possible paths for receiving "a master's degree in the subject" that does not include a teaching certificate. The M.Sc. degree in mathematics is awarded by the department of mathematics at universities; the M.A. degree in science education with a specialization in math is awarded by the department for science education at universities; and the M.Ed. in math education is awarded by the academic colleges for teacher training. In addition, the Council for Higher Education has given its provisional approval in recent years[19] for a master's degree (M.Teach) in teaching a particular subject that also includes a teaching certificate in the subject and allows teaching in grades 11 and 12, as well as a more general M.Teach degree that allows teaching only in grades 8 to 10 with a corresponding teaching certificate.

The Council for Higher Education approves the various programs for academic degrees and teaching certificates in accordance with the proven abilities of the institutions (the level of the faculty and the teaching environment) and the detailed curriculum of courses in the program.

A bachelor of science (B.Sc.) degree in mathematics at the universities includes 80 to 120 credit points[20] of math courses (40 to 60 hours) in a single-major track, out of a total of 115 to 135 credit points for the degree. Most of the universities also offer a dual-major track requiring math study of 40 to 80 credit points (20 to 40 hours). The universities have the freedom to decide the mix of courses required for receiving the degree.

The programs of study offered at all of the academic institutions (including the universities) that offer a teaching certificate are required to comply with the "guidelines in training for teaching" approved by

The programs of study offered at all of the academic institutions (including the universities) that offer a teaching certificate are required to comply with the "guidelines in training for teaching" approved by the Council for Higher Education, as recommended by the Ariav Committee.

[19] The Council for Higher Education's decision on June 30, 2009 (in Hebrew): http://che.org.il/?decision=%d7%94%d7%97%d7%9c%d7%98%d7%aa-%d7%9e%d7%9c%d7%92-37-%d7%aa%d7%a9%d7%a1%d7%98

[20] A credit point is generally awarded for one weekly hour of academic study over the course of one semester. 2 credit points equal 1 hour of study throughout the academic year.

the Council for Higher Education,[21] as recommended by the Ariav Committee. Under these "guidelines," frameworks of training were defined for teacher certification according to the "serial model" (for those who have completed an undergraduate degree) and the "parallel model"[22] (as an integral part of an undergraduate degree in education). In addition, other formats of teacher training have also been defined: an integrative framework in graduate studies in education or in discipline-specific education (M.Teach), and in special frameworks, such as expanding certification for particular population groups.

According to the guidelines, a person who has an undergraduate degree in mathematics and wishes to receive a teaching certificate in the subject is required to complete a total of 26 hours of discipline-specific studies.

According to the guidelines, a person who has an undergraduate degree in mathematics and wishes to receive a teaching certificate in the subject is required to complete, if necessary, a total of 26 hours of discipline-specific studies. (As noted, in some of the universities, it is possible to receive a degree in mathematics with only 20 hours of math studies). At least 18 hours of studies in education are also required, including at least 10 hours of theory, with the remaining hours devoted to practice teaching.

The basic outline ("the Ariav outline") for a bachelor's degree in the integrative framework with a teaching certificate includes[23] a total of 90 to 96 hours, including 48 to 60 hours of discipline-specific studies, as standard in the system of higher education in Israel for an undergraduate degree. Studies for teacher training includes 15 to 21 hours of theoretical studies in education and pedagogy (which includes at least: education studies – 4

[21] http://www.che.org.il/template/default.aspx?PageId=206 (in Hebrew)

[22] This integrative framework is an academic degree program that combines teacher training throughout the course of studies as a single package. Only in exceptional cases is it possible to receive a teaching certificate without the degree, or the degree without the teaching certificate.

[23] Decision by the Council of Higher Education (No. 1084/10) on November 21, 2006 on "Guidelines for Training for Teaching in Institutions of Higher Learning in Israel" – Ariav Committee Report.

hours; pedagogy and methodology in teaching the subject – 4 hours; research literacy in education and teaching – 2 hours) and practical experience in the education system (5 to 15 hours). In addition to discipline-specific studies and pedagogical training, the outline mandates at least 6 hours of basic studies and enrichment (2 hours of studies in the language of instruction to the level of "exempt," 2 hours of English to the level of "exempt," 1 hour of computer literacy to the level of "exempt," 1 hour of study of culture and heritage). The outline also allows for expanding the scope of basic and enrichment studies by an additional 6 hours.

In addition to the standard tracks for training teachers at the universities and colleges, the Ministry of Education offers graduates a career retraining program that follows the guidelines for teacher training, as well as a shorter program.[24]

B) Teacher training in Israel – the situation in practice

The total number of teachers in 2010 was about 125,000.[25] About 3,700 were math teachers in the middle schools[26] (2,750 in Hebrew education and 900 in Arab education), and about 5,000 were math teachers in high schools[27] (4,100 in Hebrew education and 900 in Arab education).

Data from the Central Bureau of Statistics indicates that the percentage of math teachers with an academic degree continually rose during the years 1996 to 2009,[28] reaching 93% in high schools and a similar percentage in middle schools.

Data from the Central Bureau of Statistics indicates that the percentage of math teachers with an academic degree continually rose during the years 1996 to 2009.

[24] The Teach For All program: http://tfi.org.il/Pages/Chotameducationaltraining.aspx (Hebrew)
[25] From the Ministry of Education's website: http://cms.education.gov.il/EducationCMS/Units/Owl/Hebrew/UvdotNetunim/netunim/Netunim2010.htm (Hebrew)
[26] The number of teachers in middle schools is based on 2010 data from the Central Bureau of Statistics.
[27] The number of teachers in high schools is based on 2009 data from the Central Bureau of Statistics (2010).
[28] It should be noted in this context that an academic degree and teaching certificate are now a mandatory condition for joining the *Oz Letmura* program, though the degree does not have to be in education.

The percentage of teachers with a master's degree reached 34% in middle schools at the end of this period (39% in Hebrew education and 17% in Arab education), and 41% in high schools (44% in Hebrew education and 25% in Arab education).

The Ministry of Education and the Central Bureau of Statistics do not have complete data on the fields in which teachers received their degrees

However, the Ministry of Education and the Central Bureau of Statistics do not have complete data on the fields in which teachers received their degrees.[29] Partial data on the content of math teachers' education was available in the past through self-reporting by teachers on questionnaires from the Education Ministry or via the TIMSS exams. Additional information on math teachers was recently linked to the data on higher education in Israel: information on about 70% of high school math teachers who are 45 years old or younger (who comprise about 48% of high school math teachers) and about 60% of middle school math teachers who are 35 years old or younger (who comprise about 28% of middle school math teachers). In this analysis, data is missing primarily from older teachers and from teachers who are new immigrants; this data is not available for analysis by the Central Bureau of Statistics (CBS, 2010).

The Trends in International Mathematics and Science Study (TIMSS)[30] includes data on the achievements of a sample of 3,300 Israeli students in 150 eighth-grade classes in 2007, as well as information collected about the students' math teachers via self-reporting by teachers on questionnaires. The study refers to the percentage of students (in the total population of students) whose teachers have particular characteristics and does not directly refer to the percentage of these teachers within the entire teacher population. According to the study, about two-thirds of the students tested

[29] "Teaching Mathematics in the School System – Situation Report Regarding Teachers." Submitted to the Knesset Education, Culture and Sport Committee by the Knesset Research and Information Center, March 2008.

[30] http://cms.education.gov.il/NR/rdonlyres/CD0AD5CD-0E5E-4278-BACF-6F443B38150D/104162/TIMSS_2007_fullreport1.pdf (Hebrew)

study with teachers who have an undergraduate degree (including a B.Ed.) and about a third study with teachers who have advanced degrees. Some 35% of the students study with teachers who specialized in mathematics but not the teaching of mathematics; 17% studied with teachers who specialized in teaching mathematics, but not in mathematics; and 40% study with teachers who specialized in both mathematics and the teaching of mathematics. Only 8% of the students who participated in the exam study with teachers who did not specialize in either mathematics or in the teaching of mathematics.

An analysis of the academic education of high school math teachers (an analysis conducted in 2009 based on partial data from the Central Bureau of Statistics) indicates that among teachers who are 45 years old or younger, 40% have a bachelor's degree in mathematics or in math education; about 18% have a bachelor's degree in a related field of science (engineering, computer science, statistics or physical sciences), while the undergraduate degree of the other teachers are in fields far removed from mathematics. Partial data on about 60% of middle school math teachers indicates that 42% of them were trained to teach mathematics, 4% were trained for general teaching, and 54% were trained in a different field.[31]

An analysis of the academic education of high school math teachers indicates that among teachers for whom there is data, 40% have a bachelor's degree in mathematics or in math education; about 18% have a bachelor's degree in a related field of science, while the undergraduate degree of the other teachers (about 42%) are in fields far removed from mathematics.

In addition, this analysis points to a drop in the percentage of high school math teachers who have a university degree, from 74% in 1999 to about 62% in 2009, and an increase in the percentage of graduates from academic colleges for teaching, from 24% to 34%. (During these years, no significant change was noted in the small percentage of teachers who received an undergraduate degree from a general academic college.) In the

[31] "Teaching Mathematics in the School System – Situation Report Regarding Teachers." Submitted to the Knesset Education, Culture and Sport Committee by the Knesset Research and Information Center, March 2008.

middle schools in 2009, about 34% of the math teachers were university graduates and some 61% were graduates of teachers colleges.

Trends: During the years 1996 to 2009, the number of math teachers in high school grew by 58%, in parallel to a 41% increase in the number of hours of instruction in the subject (CBS, 2010). Nonetheless, it seems that this growth trend has halted in recent years and even reversed: The percentage of new teachers in the subject declined from 16% in 1996 to a level of 9% in 2009 in the Hebrew education system, while the percentage of math teachers leaving the teaching profession rose during these years from 8% to 12%. A similar picture is reflected in the Arab education system.

Despite the steep growth in the number of math teachers in high school during the past decade, only about 40% of the new teachers were actually new in the system: 60% of the new math teachers came from within the system – half of them from other subjects in high school, and half from lower stages of education.

Despite the steep growth in the number of math teachers in high school during the past decade, only about 40% of the new teachers were actually new in the system: 60% of the new math teachers came from within the system – half of them from other subjects in high school (40% from math-related subjects and 60% from unrelated subjects), and half from lower grades in the education system (36% from middle schools – 70% were previously math teachers; and 13% from elementary schools – 45% were previously math teachers). Among the new teachers in the system in 2010, only about 29% had a degree in mathematics or were math students in the past. Among the new teachers who came from within the system, only about 25% of those who had previously taught other subjects in high school had a degree in mathematics, and only about 55% of the teachers who came from the middle schools or elementary education had a degree in mathematics or a B.Ed. with a specialization in math.

The relatively small number of new teachers in the system reflects the decline in recent years in the number of recipients of teaching certificates in mathematics from the universities and in the number of those entering the teaching profession – only a few dozen each year. In 2008, for example, only 20 math graduates with a teaching certificate from a university entered the teaching profession. The number of recipients of teaching certificates in mathematics at colleges rose only slightly during these years, and reached about 300 in 2008. Only about 63% of them actually took a teaching job in schools (including elementary education).

> **In summary**: There is a requirement in Israel for a bachelor's degree in the subject as a condition for teaching mathematics in grades 7 to 10, and a requirement for a master's degree as a condition for teaching in grades 11 to 12. There is great diversity in the training programs for undergraduate and graduate degrees in the field, and in the math knowledge provided to the graduates of these programs. The Ministry of Education and the Central Bureau of Statistics (CBS) do not have complete information on the teachers' educational background; however, an analysis of CBS data indicates that many of the teachers lack the required education.

4.3 Training institutions

4.3.1 The question of the institutions

The question of the institutions that are authorized to train future teachers in general and math teachers in particular is one that includes professional, political, social, economic and other aspects. The two main possibilities are: training teachers in the framework of research universities and training teachers in the framework of institutions dedicated to this purpose.

The two main possibilities are: training teachers in the framework of research universities and training teachers in the framework of institutions dedicated to this purpose.

Institutions dedicated to teacher training are generally characterized by a coherent program of studies whose components are all devoted to teacher training, by a faculty focused on this objective, by closer interaction with the field, and by some extent of supervision by the Ministry of Education.

Teacher training at research universities is usually characterized by the institution's greater autonomy in defining the program of studies, by a need to include in the program elements that usually involve other departments, and by the possibility of acquiring academic knowledge from researchers in the specific discipline.

All of the training programs for teachers include four components: studies in education, academic studies of a field of knowledge, methodology of a field of knowledge, and practical training. The centrality of practice teaching and the inherent tension between the expectation that the training program is relevant to practical work in the classroom and focuses on developing teaching skills, on the one hand, and the requirements of the academic world, on the other hand (Moon, 2003) have made it incumbent upon the training institutions to develop collaboration between the university and the field (for example, see Jaworski, 2010), similar to the cooperation between medical faculties and university hospitals. However, in research conducted in the United States, it was found that there is still a clear need to improve the practical experience and the connection with schools in the framework of practical teacher training in research universities:

> Although there are many college and university programs where faculty do take teacher education seriously and do a good job, there is still much work to be done within research universities in the preparation of teacher educators and with regard to rewarding faculty in higher education institutions generally for doing high quality work in educating teachers and in working with schools. (Zeichner, 2006, p. 335)

4.3.2 Training institutions in various countries

The training of secondary school teachers is conducted in the framework of universities in most countries. However, in some countries the governments reserve for themselves, in one way or another, the ability to influence the training through teaching licenses, licensing exams, and so on. This governmental policy creates some tension and increases the centralized aspect of training. In 12 European countries, there no central control over the program of teacher education, and the training institutions are autonomous in determining all of the details of the programs for training math teachers. In 18 other countries in Europe, there are various degrees of central guidance, including lists of required content for training. In England, for example, there is a central definition of the mathematical knowledge specific to teaching that should be a part of the training. But in most countries the academic institution is free to determine the ratio of mathematical content knowledge versus mathematical teaching knowledge (Eurydice, 2011).

The training of secondary school teachers is conducted in the framework of universities in most countries. However, in some countries the governments reserve for themselves, in one way or another, the ability to influence the training through teaching licenses, licensing exams, and so on.

Finland and Japan, which rank high on the global list of achievements on PISA tests in mathematics, are examples of countries in which there are various levels of central control in training teachers. In Finland, the training of teachers in recent decades has been conducted exclusively in universities, and not in teachers colleges as in the past. And the overwhelming majority of the teachers have graduate research degrees. In Japan, both possibilities are offered and the teachers are trained at universities or at institutes dedicated to training teachers (OECD, 2011).

Many training programs are offered under the umbrella of a single academic institution (a university or a teachers college), though students are required to take courses in both the disciplinary departments and in the departments of education. In other cases, the program is conducted entirely in one department for teacher training (OECD, 2005). Each of these possibilities has advantages and disadvantages. Studying in a disciplinary department helps the student attain a high level in the field of knowledge and keep up-to-date with the research, but it also makes integrative study of both the field of knowledge and its teaching more difficult. Moreover, studying in separate departments makes it harder to develop the teachers' professional identity (Calander, 2003 in: OECD, 2005). Recently, there has been growing

Recently, there has been growing recognition of the fact that the training of teachers at research universities is not only the purview of the Faculty of Education. Rather, the disciplinary departments should also be involved.

recognition of the fact that the training of teachers at research universities is not only the purview of the Faculty of Education. Rather, the disciplinary departments should also be involved (Zeichner, 2006).

In Singapore, whose students' scores on the PISA tests are among the highest in the world (Shleicher, 2011), all of the teachers receive their training in the one national institute for education. In this institute, there is no division between disciplinary departments and the school of education. Thus, it is possible to conduct a coherent program of training that is completely aimed at training teachers, at ensuring appropriate pedagogical emphases, and at refraining from conflicts of interests that frequently develop among the various departments (for example, the Education and the Mathematics departments) that share the training of teachers in universities in the world (OECD, 2011).

4.3.3 Training institutions in Israel

There are differences between teacher education tracks in universities versus colleges in Israel.

The institutions for teacher training in Israel operate as non-profit organizations or as independent companies, under the supervision and funding of the Ministry of Education's division for teacher training. The Council for Higher Education is responsible for the academic content in these institutions. Therefore, a program of study approved by the Ministry of Education is a condition for receiving funding. The ministry sets requirements pertaining to the scope, content and program of studies in the basic areas of study: Hebrew, Arabic, Judaism, heritage and culture, security, safety and first aid, study trips, and English studies. The program is also sent to the relevant ministry supervisor in order to receive an assessment of its suitability for the education system. After approval of the full program by the Ministry of Education, it is sent to the Council for Higher Education for its approval.

On the other hand, the Council for Higher Education is solely responsible for the supervision of the university tracks. The council's general division for supervising the universities is responsible for this, while a different division supervises institutions of teacher training. The Ministry of Education has argued that this system of supervision creates a situation in which the teachers trained at the universities do not meet the criteria set by the ministry, because these criteria are only enforced vis-à-vis the teacher training institutions that are subject to the supervision of the Ministry of Education and the division for teachers at the Council for Higher Education. The Ministry of Education also contends that the universities are not teaching core subjects that are essential for teachers, but that the ministry lacks the authority to order the universities to change their programs of studies.

In January 2005, the National Taskforce for the Advancement of Education in Israel (the Dovrat Committee) submitted the "National Program for Education" (the Dovrat Report) to the government. One of the report's recommendations is to significantly reduce the array of institutions for teacher training and to place it under the full responsibility of the Council for Higher Education and the council's Planning and Budgeting Committee. The government adopted the Dovrat Committee's recommendations, but implementation of the decision encountered many difficulties, and it seems that the colleges have remained under the dual responsibility of the Ministry of Education (supervision and funding) and the Council for Higher Education (responsibility for academic content).[32]

The government adopted the Dovrat Committee's recommendations, but implementation of the decision encountered many difficulties, and it seems that the colleges have remained under the dual responsibility of the Ministry of Education (supervision and funding) and the Council for Higher Education (responsibility for academic content).

[32] The website of the Division for Teacher Training, Ministry of Education (Hebrew): http://cms.education.gov.il/EducationCMS/Units/HachsharatOvdeyHoraa/Odot

4.4 The training models: parallel and serial

In general, it is possible to identify two types of models for training teachers:

The parallel model: According to this model, discipline-specific subjects are studied in parallel to the study of education and the training of teachers. Most countries operate a parallel model for middle school and high school teachers. In some of the countries (Belgium, Canada, Greece, Hungary, Ireland, Italy, Japan, Korea, Turkey and the United States), this model is the primary one for training high school teachers (OECD, 2005, p. 103).

The advantage of this model is that it allows for study of a field of knowledge, while concurrently combining this academic study with training and practical experience in teaching this subject. The disadvantage of the parallel model is that it is generally less flexible than the serial model because the students must decide to pursue teaching at an early stage of their academic studies. The parallel model might also make it harder to participate in a teacher training program after already completing a degree in a field other than education – harder or more expensive (though there are countries that award some credit for studies in other fields).

The serial model: According to this model, a program for professional training in pedagogy and teaching comes after the completion of a bachelor's degree in a field of knowledge studied in schools. This model is more common in training secondary school teachers than in training elementary school teachers. It operates in Denmark, France, Norway and Spain. In other countries (Austria, Australia, the Czech Republic, England, Wales, Finland, Ireland, Israel, Holland, Northern Ireland, Scotland, Slovakia and Sweden) both the parallel model and the serial model are offered for training teachers for secondary school education (OECD, 2005, p. 104).

The serial model provides for flexible entry into teacher training and enables the student to decide to enter teaching upon completion of a degree, while also taking into account the situation in the job market. However, there is much less integration between the field of knowledge and teaching in this model.

> **Today the consensus is that countries benefit from offering both models.**

The parallel and serial models are generally aimed at different populations of potential teachers and are expected to have different effects on the professional training of the teachers. Today the consensus (OECD, 2005) is that countries benefit from offering both models. The parallel model is attractive to students who are deeply committed to choosing teaching as a career. The serial model enables deferral of the decision to an appropriate time for the candidate, or until the candidate has enough information to make a sound decision about whether a teaching career is the right choice for him or her.

Both models exist in Israel: The parallel model exists in the framework of training math teachers in the teachers colleges and in some of the universities. The serial model is common at the universities, where a third-year math student can begin studying for a teaching certificate and, upon completion of his or her studies, receive both a bachelor's degree in mathematics and a teaching certificate in this field. (That is, the student does not receive an academic degree in math education.)

The serial model also enables a retraining framework for graduates interested in teaching. Those with a bachelor's degree in mathematics can join one of the retraining programs offered at teaching colleges or at universities that have adopted this program. In this way, they can complete didactic and pedagogic studies of mathematics. Graduates in other fields of knowledge must complete math studies.

4.5 Recommendations

Following the description in the previous chapter of the components of knowledge required for those engaged in teaching mathematics in secondary schools, the committee proposes a general outline for the training program for teaching math in secondary school education, while addressing three of the components of knowledge required for teaching mathematics – mathematical content knowledge, pedagogical knowledge and practical knowledge.

In the committee's view, comprehensive and profound knowledge in these fields mandates studies at the level and scope of a master's degree. The committee recommends developing, in the framework of direct studies for a master's degree, a challenging training program that will serve as a model of suitable training for teaching math. In the framework of the master's degree, the committee recommends research experience in math education, a deepening of mathematical and mathematical-pedagogical knowledge, and a choice of various sub-specializations.

In order to train math teachers at the required level and scope for the secondary education system, all of the relevant institutional resources must be mobilized. In this context, the committee recommends examining the possibility of developing inter-institutional academic programs and awarding a combined degree from two institutions. In this scenario, one central entity would have to assume responsibility for all of the components of the program, for ongoing collaboration and sufficient integration between the courses in the field of knowledge and courses in teaching this subject and practical training.

4.5.1 A direct track for a master's degree

The committee recommends opening a special four-year path of studies[33] leading directly to a master's degree in the teaching of mathematics and including a teaching certificate. In order to attract highly talented students, this track should offer good conditions and generous scholarships. In return, graduates of this program would commit to teach a number of years in the education system after completing a year of practice teaching. The proposed track is similar in structure to the Revivim program at Hebrew University, designed to train teachers in Jewish Studies.

The track would include basic studies for an undergraduate degree in mathematics, as specified below, in addition to more advanced courses in mathematics and math education. Extra investment in this unique track would enable courses to be developed that provide special knowledge for

[33] Without a thesis. The committee recommends allowing those who are interested to write a thesis (which is a prerequisite for a doctoral degree) during a fifth year.

teaching mathematics with emphases that are not conventionally included in academic math instruction (history, literacy, mathematics for teaching, etc.) – all this, in addition to the scope of math studies required for a bachelor's degree in mathematics. At the end of the third year, the students would receive a bachelor's degree in mathematics.

In the framework of the master's degree, the student could choose a secondary specialization – for example, teaching gifted students, teaching struggling/outstanding students, teaching in classes with students of diverse cultural background, and so on. In any case, the studies would include practice in conducting research on a limited scale in the field of math education in secondary schools or in a mathematical field.

The track would include basic studies for an undergraduate degree in mathematics, as specified below, in addition to more advanced courses in mathematics and math education, with emphases that are not conventionally included in academic math instruction (history, literacy, mathematics for teaching, and so on.).

4.5.2 Program of studies for a bachelor's degree

The committee proposes a general outline for training programs for teaching mathematics in secondary education, while addressing the various components of knowledge required for teaching mathematics, as presented in chapter 3 – mathematical content knowledge (general mathematical knowledge and mathematical knowledge for teaching), pedagogical knowledge (specific to mathematics and general), and practical knowledge, as specified below.

In general, the committee recommends continuing to conduct the training according to the guidelines for teacher training (the Ariav outline). However, the committee believes that the training for

In general, the committee recommends continuing to conduct the training according to the guidelines for teacher training (the Ariav outline). However, training for teaching mathematics in secondary education should not be allowed in conjunction with another field in the framework of a double major.

teaching math in secondary education is equivalent to a double-major program in mathematics and the teaching of mathematics. Therefore, training for teaching mathematics in secondary education should not be allowed in conjunction with another field in the framework of a double major.

The committee does not deem it appropriate to recommend a more detailed structure for the training program, and prefers to allow the institutions the academic freedom to structure a syllabus that reflects the fields of knowledge specified in this chapter, while integrating required fields of knowledge, on the one hand, and avoiding overlap and unnecessary repetition, on the other hand. Thus, the categories of teachers' knowledge presented in Chapter 3 should guide the development of syllabuses that are not necessarily identical in terms of the list of courses, their order, and so on.

A) Mathematical content knowledge

As presented in Chapter 3, it is possible to distinguish three categories of mathematical content knowledge:

A1. General mathematical content knowledge, as studied in the framework of undergraduate studies in mathematics

A2. Mathematical content knowledge related to the math content of instruction

A3. Mathematical content knowledge in various contexts

We will address each of these categories separately.

A1 | **General mathematical content knowledge**

The committee recommends that math teachers in high school have a degree based on mathematics at the B.Sc. level. Consequently, the committee chose a threshold framework of dual-major studies (with mathematics as one of the majors) as the minimum scope of math studies, so that the degree will include a math component that is no smaller in scope that that which is required in any other dual-major track.

The study of mathematics is largely built layer by layer. In fact, it is impossible to take almost any course before completing the "first-year package," including infinitesimal calculus and linear algebra. These courses are a prerequisite for any other course (with some exceptions). In addition, these are the courses in which the greatest emphasis is placed on the mathematical method, and from this perspective their importance extends beyond the specific content studied there. In light of all this, these courses must be the cornerstone of the mathematical part of the teacher's training.

The committee recommends that math teachers in high school have a degree based on the study of mathematics at the B.Sc. level. Consequently, the committee chose a threshold framework of dual-major studies (with mathematics as one of the majors) as the minimum scope of math studies in the degree. In light of all this, the infinitesimal calculus and linear algebra courses must be the cornerstone of the mathematical part of the teacher's training.

Beyond these courses, the committee does not deem it appropriate to explicitly recommend the set of additional math courses the teachers should study, and believes this decision should be left for the institutions and for the students to choose. Nonetheless, the committee notes several courses that can be of particular benefit for teachers:

1. Discrete mathematics (a component of the curriculum in high school)
2. Probability (a component of the curriculum in high school)
3. Regular differential equations (which serve as the basis for formulating many physical laws)
4. Algebraic structures (which provide a more general context for algebra studies and include some of the beautiful "pearls" of the mathematics studied for an undergraduate degree)
5. Geometry (a subject directly related to the curriculum)
6. The history of mathematics (which provides an overview of math studies and an enriching perspective on the content studied in high school)

7. Complex functions (complex numbers studied in high school)

It should be noted that the linear algebra and infinitesimal calculus courses are offered in many institutions in separate versions – for math students and for students of physics, science or engineering. The courses of the second type are more practical-oriented and are closer to the way these topics are addressed in high school. Therefore, it was ostensibly possible to determine that these courses are more suitable for teachers. However, such a decision would be tantamount to a decision that it would be best for math teachers to study science rather than mathematics. The committee recommends preferring the courses designated for mathematics because in these courses, as opposed to the parallel courses for science or engineering students, special emphasis is placed on methodological aspects and there is an in-depth examination of concepts such as "definition," "argument," "proof" and "example." In addition, the math courses should be preferred because without this foundation, the students will experience difficulty in more advanced courses.

A2 | **Mathematical content knowledge related to the math content of instruction**

Mathematical content knowledge related to the math content of instruction includes knowledge of mathematics that is accessible to school students and is not included in the program of studies. In this framework, the committee recommends including courses such as topics in number theory (prime and composite numbers, etc.), Euclidean geometric theorems – plane and space, analytic geometry, a course on prominent mathematicians and their work, and so on.

Mathematical content knowledge related to the math content of instruction also includes elementary school math topics, as well as an advanced perspective on mathematical content studied in secondary school. In this framework, the committee recommends including, for example, courses on elementary functions, E and L functions, cone sections, various systems of coordinates, non-Euclidean geometries and connections between algebra and geometry.

A3 | **Mathematical content knowledge in various contexts**

The committee recommends including courses in the training of math teachers that address related fields of knowledge that are deeply linked to mathematics, including physics in particular. We also recommend adding a course on mathematical models that describe natural phenomena in general, as well as courses that focus on mathematical aspects of economics, earth science, computer science, art and more.

The committee recommends including courses in the training of math teachers that address related fields of knowledge that are deeply linked to mathematics, as well as courses that explore different ways of developing math literacy.

The committee recommends including courses in the training that explore different ways of developing math literacy; solving complex, multidisciplinary and text-heavy problems; research tasks; strategies for solving problems; and recognizing various types of mathematical argumentation.

B) Pedagogical knowledge

B1 | **General pedagogical knowledge**

The committee recommends that the part of training that focuses on general pedagogical knowledge should include courses in the philosophy of education, in social and cognitive psychology, in the psychology of the adolescent, in learning disabilities[34] and in class management, as well as courses that focus on research and research methods in math education.

[34] In this context, the development of research in cognitive neuroscience should be noted. This research is relevant to the field of teaching and math education. The committee's impression is that the developing research on dyscalculia and number perception is likely to become an important component in the training of teachers.

B2 | Unique pedagogical knowledge

The committee recommends devoting a significant place in the training to the pedagogical knowledge unique to the field of content, including all of its four components.

The committee recommends devoting a significant place in the training to the pedagogical knowledge unique to the field of content, including all of its four components:

(I) Knowledge of mathematics for teaching

In this category of knowledge, courses on how to teach specific fields of mathematics should be included, such as teaching algebra and teaching geometry. These courses should address the ways students think, the erroneous perception of concepts in the field, the typical levels of thinking, the different representations of concepts in the field, the analysis of mathematical ideas that appear in this field, the analysis of classroom incidents relevant to the subject, and so on.

(II) Didactic mathematical knowledge

Two types of courses should be included in this category of knowledge. One type of course should deal with general issues in teaching mathematics, such as the goals of instruction, teaching math in a heterogeneous classroom, the role of mathematical discourse, instilling quantitative thinking, psychological and cognitive aspects of mathematics, intuition, justification, rational and conceptual understanding, gender, teaching struggling students and those with learning disabilities in mathematics, and teaching gifted students in mathematics. The second type of course should focus on fundamental planning of units of study, including the planning of various methods for teaching the subject, prior knowledge required for teaching, adapting the material for various populations of students, using teaching methods that can contribute to the learning process, low and high orders of thinking, planning evaluation methods for each unit, determining the unit's place in the curriculum, teaching the unit's content in a technological environment, associated curricular resources, and so on.

(III) Curricular knowledge

The curricular area should be included in a course addressing the program of study in middle schools and high schools, the objectives of teaching and their rationale, and the division of the curriculum to topics and levels. There is room here to also address the curriculum in elementary schools and in institutions of higher education, in the spirit of "know where you're coming from and know where you're going." In addition, there is room to examine alternative curricula used in other countries.

(IV) Knowledge about evaluating learning and achievements

A course focusing on evaluation and learning should take into account unique aspects of assessment in mathematics. It should address a variety of ways of evaluation, the composing of exams in mathematics, relevant taxonomies of learning, evaluation tests (national and international) and ways to combine all of this in teaching, while maintaining a close connection to the actual work of teaching in math class.

In addition to all of the above, student teachers require good knowledge in the Hebrew language and a foreign language (English), as well as knowledge in citizenship and national heritage. The committee recommends allowing prospective math teachers to also study a number of courses on general, non-mathematics subjects (physics, philosophy, art, literature, history and so on).

C) Practical knowledge

Practical knowledge and experience comprise a central component in the training of math teachers. For this purpose, the committee recommends including practical experience in each year of study, with an emphasis on the connections between academic knowledge and practical knowledge.

The students should participate in guided observation of math lessons in various classes, and should acquire experience in planning study units, in

Each of the students should be allowed to teach part of the math curriculum to a group of students on a regular basis, while assuming full responsibility for planning the lesson, teaching it, assessing achievements, reporting to the school, participating in staff meetings, and so on.

In addition, the students should experience one-on-one work with struggling students and outstanding students.

teaching colleagues, in individual instruction and in teaching a lesson to a small group of students. Each of the students should be allowed to teach part of the math curriculum to a group of students on a regular basis, while assuming full responsibility for planning the lesson, teaching it, assessing achievements, reporting to the school, participating in staff meetings, and so on. In addition, the students should experience one-on-one work with struggling students and outstanding students. Didactic guidance from a skilled teacher of teachers should accompany these experiences in an ongoing way.

In regard to the demand for a bachelor's degree in mathematics, which appears in recommendations 4.5.1 and 4.5.2, see a minority opinion in the introductory chapter.

4.5.3 Additional recommendations

A. The committee calls for finding ways to increase the demand for the teaching profession so that it will possible to set high standards for acceptance to teacher training and to choose the best candidates.

B. This chapter is based on data pertaining to the population of math teachers in Israel. Unfortunately, this data is only partial. The committee recommends making an effort to build a full and up-to-date database.

Chapter 5
Licensing Examinations

5.1 Introduction

Teachers are the most influential school-based factor in students' achievements. Therefore, improving the effectiveness and parity in schools depends to a great extent on having highly skilled teachers who are motivated to do their utmost and who are equipped with the requisite resources. Consequently, a policy aimed at raising the level of the teachers' performance has the greatest chance of significantly boosting the students' learning (OECD, 2005). Evaluating a teacher's performance is considered a central tool for continually improving the effectiveness of teaching because it highlights the teacher's strong characteristics as well as those which require improvement. From this perspective, institutionalizing an evaluation system is an essential step in the effort to improve the effectiveness of instruction and learning, and in raising educational standards. Licensing examinations can serve as a central tool in this system.

Institutionalizing an evaluation system is an essential step in the effort to improve the effectiveness of instruction and learning, and in raising educational standards. Licensing examinations can serve as a central tool in this system.

5.2 Goals and objectives for evaluating the professional performance of math teachers

There are usually two principal goals in evaluating a teacher's professional performance. One is the function of improvement – improving teaching activity by identifying the strong and weak points in this activity, and making it the focus of professional development. The second is the function of accountability – ensuring the optimal activity by teachers for promoting the students' learning.

There are usually two principal goals in evaluating a teacher's professional performance. One is the function of improvement – improving teaching activity by identifying the strong and weak points in this activity, and making it the focus of professional development. The second is the function of accountability – ensuring the optimal activity by teachers for promoting the students' learning.

The function of improvement: Evaluating the performance of teachers for the goal of improvement focuses on providing effective practical feedback for improving teaching through professional development. Such feedback helps teachers to develop awareness of their performance, to conduct reflection on their activity inside and outside of the classroom, and to improve their performance. Usually, the evaluation is conducted according to the school context, so that the teacher's opportunities for professional development are adapted to the school development program.

The function of accountability: Evaluating teachers for the goal of accountability focuses on assigning the responsibility for their performance on the teachers themselves and in establishing a connection between their performance and a range of ramifications for their career. The goal of this type of evaluation is to offer remuneration to encourage optimal performance by teachers. This generally includes incentives based on high performance – career advancement, a salary increase, bonuses, etc. – or sanctions in the case of low performance. In essence, the evaluation of teachers for the goal of accountability summarizes the teachers' performance, and is usually conducted at various junctions of their career. This type of evaluation could also be used in awarding a teaching license.

The tension between the function of improvement and the function of accountability: Combining the two functions (improvement and accountability) in evaluating teachers in one all-encompassing assessment poses difficult challenges. When the evaluation is intended to improve the work conducted in the school, the teachers are generally more willing to expose their weaknesses, with the expectation that exposing such information will lead to more effective decisions regarding their training and development needs. On the other hand, when the teachers face the potential repercussions of the evaluation results on their career or salary, there is less readiness to expose weak aspects of their performance and, consequently, this jeopardizes the fulfillment of the function of improvement. Furthermore, the use of the same evaluation process for two objectives undermines the utility and effectiveness of some of the assessment tools (such as self-assessment) and creates an additional burden for the evaluators because their decisions will have contradictory implications to some extent. In other words, tension is created between improving performance (by identifying points of weakness) and hampering career advancement (if the evaluation prevents the teacher from advancing in his career). In fact, many countries use a special combination that includes a number of objectives and methodologies (Stronge & Tucker, 2003). Other pitfalls include a lack of maturity in teacher evaluation – for example, when the evaluation is not implemented, when the evaluators and the evaluatees have little experience, or when the legitimacy of the evaluators is not recognized.

Combining the function of improvement and the function of accountability in the same evaluation process creates difficult challenges.

To some extent, the attempt to achieve improvement through accountability creates tensions. An overemphasis on accountability is liable to generate a feeling of insecurity or fear among the teachers and lower their assessment of their work (OECD, 2009). On the other hand, the teachers and teachers' unions expect social recognition of their work and expect opportunities for professional development by establishing formative systems for evaluating teachers (Avalos & Assael, 2006).

5.3 Evaluating the professional performance of math teachers

Among teachers of mathematics, like teachers of other subjects, the quality of teaching can be assured at the stage of selecting teachers, during the course of teacher training, when beginning to work as a teacher, and throughout the teacher's career. During the first two stages, the evaluation is conducted by the training institutions. In this chapter, we will discuss the issue of evaluating new math teachers (for the purpose of licensing) and in Chapter 8 of this document we will focus on evaluation throughout the math teacher's career (for the purpose of improvement or advancement).

5.4 Evaluating for licensing purposes[35]

In important countries in the world, there are licensing exams for teachers. In these countries, the exams are not conducted on the basis of research knowledge that supports them, but rather for historical or political reasons. In Israel too, the demand for licensing and certification exams for teachers is occasionally raised due to the widespread public dissatisfaction with the education system in general and with math instruction in particular.

The main tool used by some countries to assure a minimum level of quality in teaching is a licensing examination. The licensing exam is intended to winnow out candidates who do not meet the quality requirements defined by policy makers to ensure that teaching positions are filled by suitable teachers who will contribute to improving the system and the output of learning.

In important countries in the world – most of the states in the U.S., Germany, Japan, France, and more – there are licensing exams for teachers. In these countries, the exams are not conducted on the basis of research knowledge that supports them, but rather for historical or political reasons. For example, the licensing exams for teachers in Japan and in Germany are part of a process in which teachers compete for a civil service job that accords social status and good pay. In the United States, licensing exams were instituted as a result of public dissatisfaction with the education system and, in particular, dissatisfaction with the achievements of students in the various subjects

[35] This chapter is partially based on a lecture by Prof. Miriam Ben-Peretz, who was invited by the committee, during a seminar held in collaboration with the annual convention of the National Center for Secondary School Math Teachers in December 2010. http://education.academy.ac.il/Admin/Data/Publications/mathematics-symposium-report-122010.pdf (Hebrew)

(Goldhaber, 2007). In Israel, the demand for licensing and certification exams for teachers is occasionally raised for the same reason – the widespread public dissatisfaction with the education system in general and with math instruction in particular.

Studies on the impact of certification exams on students' achievements have produced contradictory findings. (Most of these exams have been conducted in the United States.) In a 2007 study conducted in North Carolina in which hundreds of teachers and thousands of students participated, a positive correlation was found between the experience of math teachers and the achievements of their students. A similar correlation was found between the teachers' scores on certification exams and students' achievements. (Clotfelter, Ladd & Vigdor, 2007). On the other hand, the scores on the voluntary National Board Certification exams in the U.S. were not indicative of effective teaching according to the criteria defined by the researchers (Goldbaher, 2007; Cantrell, Fullerton & Kane, 2008).

Policy makers support licensing exams and regard them as a simple and relatively quick solution for the system's problems. Indeed, it is possible to list many benefits in instituting licensing exams. Success on the exams can provide official confirmation of the teachers' competencies and their suitability for the teaching profession. It can boost their self-confidence and encourage greater trust among parents and students for the teachers. Furthermore, licensing exams can contribute to the status and prestige of the teaching profession because it refutes the disparaging myth that "anyone can be a teacher." In addition, standard licensing tests can serve as a tool for assuring the quality of teacher training and for increasing the competition between training institutions in recruiting high-quality candidates and properly training them to successfully pass the licensing exams. A licensing exam that is mandatory for all prospective teachers would also ensure suitable training in alternative training programs offered (in Israel and in other countries) as a response to the shortage of teachers, particularly in the fields of mathematics and English. Moreover, licensing exams, which are an expression of the state's responsibility for the quality of those certified to teach, would relieve the Ministry of Education from having to manage input and supervise the content of studies at institutions that train teachers.

Definitive evidence has yet to be found regarding the advantage of teachers who have passed licensing exams versus their colleagues who have not passed such exams. However, there is increasing evidence of a positive, albeit weak, correlation between the licensing of teachers and the achievements of their students, particular in mathematics.

It is important to note that the contribution of licensing, as reflected in the research literature, is not unequivocal; definitive evidence has yet to be found regarding the advantage of teachers who have passed licensing exams versus teachers who have not passed such exams. However, there is increasing evidence of a positive, albeit weak, correlation between the licensing of teachers and the achievements of their students, particular in mathematics (Goldhaber, 2007; Clotfelter, Ladd, Vigdor, 2007; Kleiner, 2000). It should be noted that this weak positive connection does not lead to the conclusion that the exams are unnecessary: They are important in providing important information to the local authorities involved in hiring teachers, information that can be weighed against the candidate's other qualities.

As opposed to the many potential advantages of licensing exams for teachers, instituting these exams may entail considerable difficulties. First of all, it is difficult to introduce changes and innovations in the education system, a process that is generally conducted inefficiently. Moreover, the institutions for teacher training do not support licensing exams because they fear this would harm their academic freedom. They believe that the exams would lead to uniformity in training and in preparing for the exam, and would jeopardize the diversity and uniqueness of the various training institutions. The support of the teachers' unions for licensing exams is also not guaranteed. The Federation of Teachers announced in the wake of the report by the National Taskforce for Promoting Education in Israel (the Dovrat Committee) that it would support any measure that leads to the improvement of the school system and teachers' performance, including the evaluation of teachers. The head of the union of secondary school teachers, on the other hand, expressed doubts about the effectiveness of licensing exams and their contribution to improving the education system. There is also a monetary constraint because the development and maintenance of a system of licensing exams are complex processes that entail substantial costs.

The issue of instituting licensing exams for teachers in general and for math teachers in particular is complicated and raises numerous questions that need to be addressed, including questions pertaining to the content of the exams (What should the candidates for teaching math be tested on?), the timing of the exams (at the beginning of the teacher's professional path as a condition for entering the education system, or after several years as a condition for permanent appointment?) the nature of the exams (uniform, not uniform), the entity responsible for composing or administering the exams (RAMA – The National Authority for Measurement and Evaluation in Education, or a special entity established for this purpose?).

There are various models of licensing exams in terms of content and timing. In the United States, teachers interested in certification by the National Board for Professional Teaching Standards (NBPTS)[36] take the certification exam after four years of experience. In order to receive NBPTS certification, the teachers must pass a written test as well as a test conducted via computer simulation. The exam tests the teachers' knowledge in a field of study (mathematics, for example) and their pedagogical skills. In addition, the teachers submit a portfolio that documents their performance, including two film clips demonstrating their ability to instill skills and knowledge (30 minutes) and their ability to encourage teamwork in the classroom (20 minutes), evidence of their ability to initiate and develop projects in collaboration with the community and to maintain a good connection with the parents. The teachers also submit examples of their assessment of their students' achievements. One organization devoted to improving teacher training in the United States set two licensing exams – one for beginning teachers (which confirms their basic ability to engage in teaching), and one for experienced teachers (which is intended to confirm their advanced pedagogical skills). In Germany and France, licensing exams are governmental, and success on the exams paves the way to the sought-after status of civil servant and a high salary. In France, there is an evaluation system that conducts exams for professional advancement once every few years, and in Germany, there are exams administered during the course of the teacher's training.

[36] http://www.nbpts.org/

The National Taskforce for Promoting Education in Israel (the Dovrat Committee) recommended in 2005 that "passing licensing exams should be a condition for receiving a teaching license (as customary in other professions) and for receiving a job as a teacher. No one should be employed for more than two years (after one year of practice teaching) without passing a licensing exam" (p. 146). The National Taskforce for Promoting Education in Israel also stipulated that the Ministry of Education should be responsible for determining the standards for beginning teachers and for developing appropriate theoretical and practical exams with the help of the National Authority for Measurement and Evaluation in Education (RAMA). It was also recommended that the Ministry of Education award teaching licenses to those who pass the licensing exams. The taskforce also recommended that those who take the licensing exam should have a full academic degree (after the transition period), at least a B.A. or B.Sc. in the relevant field of knowledge, a teaching certificate and at least one year of practice teaching in a school or kindergarten. This implies that the alternative paths of training will not prepare their graduates to be examined or to enter the profession.

5.4 Recommendations

1. The committee recommends instituting a standard licensing exam for all candidates for teaching mathematics in secondary schools. The exam should be held at the end of the year of practice teaching; the results should be published within a few weeks and passing the exam should be a condition for entering the teaching profession. The exam should be governmental and uniform for all of the communities, though adapted for culture-dependent contexts. The language of the exams should be in the teacher's language of instruction (Hebrew or Arabic).

   The committee does not rule out alternative training paths, provided that their graduates are required to pass the licensing exams.

2. The committee recommends that the licensing exam assess the activity of math teaching in secondary education – that is,

the practical use of the components of knowledge described in chapters 3 and 4, while exercising educational judgment. In addition to knowledge questions, the exam should include an analysis of situations and cases from the world of teaching and schools. This combination of questions and assignments in the exam should enable shortcomings to be identified that would prevent licensing.

The licensing exams should reflect a practical ability to teach, as befits the practical nature of the teaching profession, and should not suffice with theoretical questions only. Therefore, the committee proposes that the licensing exam be administered, as noted, near the end of the year of practice teaching and that it should include a written test and an oral test aimed at providing an impression of the candidate in action (a filmed lesson, for example). It is easier to examine components such as the students' math knowledge or math comprehension, but there are also tools for examining the teaching skills of the teacher in action, and it is important to use them.

3. The committee recommends forming a committee of experts to define the content and structure of the licensing exam for math teachers and to guide its development. The task of composing the exam itself should be assigned to an entity that is independent of the training institutions, an entity that has expertise in

The committee recommends instituting a standard licensing exam for all candidates for teaching mathematics in secondary schools. The committee does not rule out alternative training paths, provided that their graduates are required to pass the licensing exams.

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assessment and in developing evaluation tools. It is important that official representatives of the teachers themselves (via their representative organizations) be involved in developing the tools. In addition, the licensing exams should be dynamic, evolving and adapting to the latest standards of the teaching profession. Only in this way will it be ensured that the exams contribute to the advancement of worthy teachers and the advancement of the profession.

4. The committee believes that research is needed to examine the connection between teachers' scores on licensing exams and the effectiveness of their teaching, which should be measured according to agreed-upon criteria – one of which is the teacher's added value for the achievements of his students. The impact of instituting licensing exams on candidate recruitment and on the training of math teachers should also be examined.

Chapter 6 Professional Development of Secondary School Math Teachers

In this chapter, we discuss the component of teacher training that comes after the teachers' professional certification (in-service training), referred to here as the "professional development of teachers." This pertains to the development that occurs throughout the math teachers' professional life, in parallel to their work in the classrooms. As in the training that precedes the teacher's entrance into the classroom (pre-service training), the processes of teachers' professional development are based on the same components of knowledge as described in Chapter 3, but this time these components will be those related and adapted to the needs of the teachers' everyday practice.

6.1 Why is professional development needed for math teachers?

The training for a teaching certificate in secondary school mathematics provides the basic tools the teacher requires in order to enter the classroom. Building expertise in any knowledge-intensive profession, and particularly in teaching, requires ongoing study and professional development throughout one's career – lifelong learning (Higher Committee for Scientific-Technological Education, 1992; Jaworski, 1994; Loucks-Horsley, Hewson, Love, & Stiles, 1998; National Council of Teachers of Mathematics, 2000; Advisory Committee on Mathematics Education, 2002; Reference Group on Teacher Standards, Quality and Professionalism, 2003; Even & Ball, 2009).

During the initial years of teaching, professional development is especially aimed at supporting teachers in classroom instruction – including planning,

During the initial years of teaching, professional development is especially aimed at supporting teachers in classroom instruction.

Subsequently, the teachers' professional training should also contribute to reinforcing and updating all of the components of knowledge (see Chapter 3 above). Furthermore, the programs for the professional development of math teachers should also enable teachers to adapt their teaching to developments and changes that occur in society, science and technology.

selecting and adapting teaching and learning materials, adapting the pace of instruction to the students, challenging and expanding their thinking, planning learning settings that provide solutions for the diversity among the students, developing appropriate assessment tools for the learning settings, and so on. The teachers learn these topics as part of their pre-service training, but they become clearer and sharper only when engaged in everyday teaching. Subsequently, the teachers' professional training should also contribute to reinforcing and updating all of the components of knowledge (see Chapter 3 above). Furthermore, the programs for the professional development of math teachers should also enable teachers to adapt their teaching to developments and changes that occur in society, science and technology. These programs should introduce teachers to opportunities for learning challenging mathematics and innovative pedagogy, and encourage them to take a new look at the mathematics they are teaching and the methods of teaching mathematics. The programs should discuss innovative approaches to teaching and how to implement them in their classrooms (Cooney & Ball, 1997; Krainer, 1996).

6.2 Goals for the professional development of teachers

Various goals derive from the aforementioned, and these goals customarily form the basis for developing programs for the professional development of math teachers. These goals are reflected in the professional literature:

- **Knowledge and expertise:** Programs for the professional development of teachers should provide ongoing support in developing the teachers' expertise, support that ensures a high quality of instruction in mathematics in the school. Such quality includes developing all of the components of knowledge, skills, flexibility, creativity and more. All these are to be developed with an emphasis on the connections between the teachers' mathematical knowledge, pedagogical knowledge and practical knowledge.

- **A learning and working community:** Programs for the professional development of teachers must support the involvement of new teachers as members of a working and learning community within the school framework or in extra-curricular frameworks; support experienced teachers as members of the communities while strengthening ties and collaboration between teachers; and support their adaptation to changes that occur in math education (changes in norms, routines, language, etc.) in accordance with the developments that occur in society, science and technology in the modern era.

- **Changes in the teaching of mathematics:** Professional development programs for teachers should support teachers who need to implement and apply new methods of teaching math in their classrooms, including new mathematical content, new methods of instruction, and the inclusion of math literacy in the study of mathematics.

- **Special populations:** Professional development programs for teachers should develop expertise in addressing special populations of students, for example: teaching students who have learning disabilities and difficulties, teaching gifted and outstanding students in mathematics, and teaching students from a different cultural background.

- **Leading roles:** Programs for the professional development of teachers should prepare teachers for leadership roles – subject coordinators, counselors, teachers of teachers, and researchers.

6.3 Guiding principles for the professional development of teachers

Various programs for the professional development of math teachers in secondary schools are differentiated from one another by how they combine and balance the goals defined above. Despite the differences between the goals of the programs, main principles can be identified and recommended for ensuring a high quality of professional training for math teachers. The literature and professional discourse point toward several guiding principles for the professional development of math teachers. In this section, we will discuss the main principles and components in the development process:

★ Developing various types of knowledge and integrating them

★ Using reflective activity

★ Fostering and coalescing a learning and working community of teachers

★ Focusing on fostering practical teaching abilities

★ Deepening knowledge in the language of instruction

★ Sensitivity and ongoing adaptation for changes in society, science and technology

We will address these principles here:

Development of various types of knowledge and integrating them

The tasks should focus the teachers' attention on challenging mathematics and help them to become more aware of the way their students think.

The professional development of math teachers should take into account the complex structure of teachers' knowledge. Activities for teachers should reflect the mathematics and the pedagogy recommended for implementation. The tasks should be challenging and combine mathematical and pedagogical subjects (Cooney & Krainer, 1996), and should focus the teachers' attention on challenging mathematics and help them to become more aware of the way their students think (Borko & Putnam, 1995). It is extremely important to maintain a close connection between what is learned in professional development and the teachers' work in the field. It is essential to ensure a connection between the teachers' theoretical knowledge and practical knowledge. In order to assimilate what they learn, teachers must implement in their work the math and pedagogy they experienced as students.

Using reflective activity

The theories of reflective practice emphasize that the reflective activity of the teacher and the student is a central means for promoting the development of thinking (Dewey, 1933; Schön, 1991; Jaworski, 1994; Krainer, 1998). The ideas of reflection-on-action and reflection-in-action have been recognized as central components that contribute to the development of teachers' knowledge and expertise (Jaworski, 1998). Consequently, programs for the professional development of math teachers should treat teachers as learners who are critically and continually analyzing and drawing conclusions about the effectiveness of their methods of teaching and the students' ways of learning.

Fostering and coalescing a learning and working community of teachers

A perception of the social nature of the learning process leads to regarding math teachers as communities of workers and learners (communities of practice) characterized by norms (social and socio-mathematical), language, routines, sensitivities and shared products created in a context of action (practice) (Lave, 1996; Lave & Wenger, 1991). On the one hand, this practice is conducted as part of a complex system of beliefs about the society, educational approaches, the program of study, evaluation and the school environment that characterizes the community. On the other hand, the teachers' perceptions about what to do and how to do it are determined by the norms of the community (Rogoff, 1990; Lave & Wenger, 1991). Thus, programs for the professional development of teachers should be based upon norms, language and routines that characterize the community.

For beginning teachers, professional development should support their involvement in the community and the adoption of norms, routines and language that can enable them to become part of the community. Programs for the professional development of experienced teachers should encourage collaboration and mutual support between teachers.

Focusing on fostering practical teaching abilities

The work of teaching includes various practices – presentation of the study material, definition of concepts, demonstration of mathematical considerations, explanation of ideas, analysis of learning materials, formulating tasks for students, facilitating discussions, asking questions, evaluating students' work, responding to students' erroneous perceptions, using students' insights to expand the lesson, posing challenges to students, and more. One of the main problems raised in the literature in the field of professional training for math teachers is the disconnection between formal knowledge acquired in various frameworks of professional development and the practice of teaching itself.

If professional training for math teachers is to contribute to the teacher's actual ability to teach, then the pursuit of formal knowledge is not enough – this knowledge must be connected to the practice of teaching.

If professional training for math teachers is to contribute to the teacher's actual ability to teach, then the pursuit of formal knowledge is not enough – this knowledge must be connected to the practice of teaching (Ball & Cohen, 1999; Even & Ball, 2009; Leikin, 2011; Gal, 2011). Such connection can be achieved in various ways, such as developing the teachers' knowledge and understanding of the students' ways of learning and thinking in the field of mathematics (Carpenter & Fennema, 1992; Even & Tirosh, 2008; Gal & Linchevski, 2010; Gal, 2011), mirroring desired mathematics and pedagogy in learning settings designed for teachers (Cooney & Krainer, 1996), highlighting the connections between mathematics as a discipline and mathematics in the schools (Even, 2011; Zazkis & Leikin, 2010), observing recorded lessons and critically analyzing them (Lampert & Ball, 1998) and more. This principle of focusing on the practice of teaching is also consistent with theoretical frameworks of learning this practice – situated learning, for example (Lave & Wenger, 1991).

Deepening knowledge in the language of instruction

Professional development should foster the precise use of language (Hebrew or Arabic) in teaching mathematics. Courses for teachers should create various opportunities to improve their mastery of the language of instruction, and should emphasize the importance of the use of correct mathematical language. A reflective discussion of the use of language in teaching mathematics should promote the teachers' understanding of the important role of language in developing mathematical knowledge and in improving the students' abilities to express themselves.

Sensitivity and ongoing adaptation for changes in society, science and technology

As noted above, math education is sensitive to the developments in society, science and technology. Therefore, programs of study in mathematics and teaching approaches undergo periodic changes. These changes include, for example, technological innovations in education or math literacy that are now being introduced into education systems in the world and into some of the international tests that evaluate the knowledge of students. Programs for the professional development of math teachers that aim to instill changes must place an emphasis on the teachers' existing beliefs and guide the learning toward changing these beliefs. In addition, it has been argued that one of the central components in the professional development of math teachers is a process of re-contextualizing what is learned in teacher training programs to what they will be teaching in the classroom (Llinares & Krainer, 2006). On the other hand, Leikin presented (Leikin, 2011) the "conviction loop" as one of the central mechanisms that make it difficult to implement what is learned in programs for the professional development of math teachers: In order to implement what is learned, teachers must be convinced of its effectiveness with students. And in order to be convinced, teachers must implement what is learned and succeed in this implementation.

6.4 Models for the professional development of teachers

The principles discussed above can be adopted in various ways, such as continuing education programs for teachers, ongoing and dedicated courses (at teachers' centers) and studies for advanced degrees. Common models include:

★ Action research (for example: Jaworski, 1998)

★ Lesson study (for example: Lewis & Tsuchida, 1998)

★ Case analysis: (for example: Shulman, 1992; Gal, 2011)

★ Cognitively guided instruction (for example: Carpenter & Fennema, 1992).

★ Teaching experiment (for example: Steffe & Thompson, 2000)

6.5 Recommendations

In Israel, there is a relatively developed array of support for professional development, and considerable resources are invested in continuing education for teachers and in providing compensation for participation in continuing education. Advanced training funds encourage teachers to study during sabbatical years and the new salary accords (*Ofek Hadash*, *Oz Letmura*) include an emphasis on professional development. In its recommendations, the committee sought to support some of the existing trends in the Ministry of Education and to strengthen other aspects in the array of professional development of math teachers in secondary education.

The committee recommends that the professional development of math teachers be viewed as an integral part of the teacher's job and work in the school.

1. Professional development of math teachers throughout their years of work (life-long learning) plays a crucial role in the success of math education. Therefore, the committee recommends that the professional development of math teachers be viewed as an integral part of the teacher's job and work in the school. Accordingly, it is necessary, on the one hand, to compensate teachers in order to encourage them to participate in professional development of various types. On the other hand, conditions should be created to significantly improve the effectiveness of the system of professional development and to increase the resources invested in this.

2. Professional development of teachers occurs during the course of their work in the school, and as a result of their participation in extra-curricular frameworks designated for this. Therefore, the committee recommends:

   a. There should be a formal system of school support for teachers' professional development (including instruction, team work, and more).

   > **Academic institutions engaged in the professional development of teachers should demonstrate responsibility for the level of teaching in the schools and offer academic-level courses that are adapted (in terms of hours, content and prices) for teachers.**

   b. Academic institutions engaged in the professional development of teachers should demonstrate responsibility for the level of teaching in the schools and offer academic-level courses that are adapted (in terms of hours, content and subsidized prices) for in-service teachers, with or without accruing credits toward a master's degree in teaching mathematics.

   c. Professional development should be encouraged throughout a teacher's years of teaching, according to a systemic and long-term view. Pursuant to this goal, higher compensation

Higher compensation could be offered for clusters of courses sharing a common denominator, such as a series of courses that awards a certificate of specialization (in teaching struggling or outstanding students, in coordinating a particular field or subject, in managing evaluation) or certification in training teachers.

could be offered for clusters of courses sharing a common denominator, such as a series of courses that awards a certificate of specialization (in teaching struggling or outstanding students, in teaching a particular field of study, in coordinating a subject, in managing evaluation) or certification in training teachers.

d. Teachers should be able to utilize the hours designated for continuing education in the framework of *Ofek Hadash* and *Oz Letmura* in order to participate in academic studies for advanced degrees and ongoing studies as part of their professional development.

3. In order to maintain a close connection between what is learned in the frameworks for professional development and the teachers' work in the field, the committee recommends:

   a. Professional development for math teachers should be encouraged in the framework of the school's teaching staff (team-based continuing education).

   b. The committee recommends offering courses that operate in collaboration between mathematicians and teachers of mathematics, if possible.

4. The committee recommends that the Ministry of Education set standards and conduct evaluation and monitoring of continuing education programs in order to facilitate the allocation of funds in accordance with the quality of the programs.

5. The committee recommends conducting comprehensive research focusing on the effectiveness of various models for the professional development of math teachers in order to enable the implementation of more effective models, as well as research on methods for developing math literacy.

6. In order to further encourage professional development, the committee recommends that studies toward a master's degree be taken into account when promoting teachers and appointing them to special positions such as coordinators of a field of study, teacher training, management and supervision.

Chapter 7
Training and Professional Development of Mathematics Teacher Educators[37]

7.1 Why are training and professional development needed for mathematics teacher educators?

The role of mentors, instructors and advisors of math teachers is to support the teachers' learning in various frameworks of professional development. Thus, mentors, instructors and advisors of math teachers are actually math teacher educators, though they are usually not referred to as such. For example: As part of the effort to institute a new program of studies in mathematics for middle schools, the Ministry of Education recruited about 200 instructors whose role is to closely advise middle school math teachers in the schools. The role of these instructors – or these teacher educators – is to help the teachers in the process of learning the new curriculum, including its new content, its different sequence of instruction, and its explicit emphases on thinking, understanding, problem solving and argumentative discourse – characteristics that are lacking in most of the math classrooms today. Moreover, with the growing awareness of the importance of math literacy and its anticipated inclusion in the math curriculum, there will also be a growing need to support teachers as they learn this type of mathematical knowledge and methods of teaching math literacy in the framework of math studies in the school.

[37] In this section, the term "teacher educators" refers to those who engage in the actual training and professional development of teachers – that is, mentors, instructors and advisors of math teachers, even though not all of them are always called teacher educators.

Those who fill the roles of mentors, instructors and advisors are usually teachers whose "reputation precedes them." But just as not everyone who knows mathematics necessarily knows how to teach this subject, not everyone who is a good teacher necessarily knows how to mentor and teach others to be good teachers. As past experience demonstrates, support for teachers' learning is not a trivial matter, and often the learning opportunities offered to teachers do not foster learning that improves the teachers' work in the classroom (Borko, 2004; Wilson, & Berne, 1999; Zaslavsky, Chapman, & Leikin, 2003). Proper training of mentors, instructors and advisors for math teachers could contribute to the creation of a professional group of teacher educators who could promote significant professional development of math teachers. This development would contribute to the improvement of teaching in practice, and thus also contribute to improving the opportunities for students to learn mathematics.

Just as not everyone who knows mathematics necessarily knows how to teach this subject, not everyone who is a good teacher necessarily knows how to mentor and teach others to be good teachers.

Proper training of mentors, instructors and advisors for math teachers would contribute to the creation of a professional group of teacher educators who could promote significant professional development of math teachers.

The question of the training and professional development of math teachers has attracted considerable attention in recent years in various countries. On the other hand, there has been little discussion (in public discourse or in the research literature) about the training and professional development of mathematics teacher educators (for both prospective and current teachers). This is despite the fact that they are actually responsible for the teachers' training and professional development. The disregard for this question is surprising when one considers the great importance attributed to the training and professional development of math teachers as a means of improving the math education offered to students. Recently, this question has been identified as a critical link in the effort to improve the training and professional development of math teachers (Even & Ball, 2009; Jaworski & Wood, 2008).

7.2 Challenges in the training and professional development of mathematics teacher educators

Three main problems can be identified when reviewing the literature in the field of training and professional development of mathematics teacher educators. The first is that there is practically no research on this subject. Contrary to the field of training and professional development of math teachers, which has attracted considerable attention in various countries during the past two decades (Advisory Committee on Mathematics Education, 2002; Even & Ball, 2009; Reference Group on Teacher Standards, Quality and Professionalism, 2003), there has been almost no discussion in the literature about the field of training and professional development of mathematics teacher educators. In a review conducted by Elliot (Elliot, 2005) on this subject, she found in the literature only three opportunities for the professional development of mathematics teacher educators, including two in the United States and one in Israel: in the U.S., in the framework of the Developing Mathematical Ideas project (Davenport & Ebby, 2000) and in the framework of the Quantitative Understanding: Amplifying Student Achievement and Reasoning project (Stein, Smith & Silver, 1999) and in Israel in the framework of the Manor project (Even, 1999). The study by Elliot was itself conducted in the United States in the framework of the Leadership Curriculum for Mathematics Professional Development project.

Additional studies recently conducted report about professional training of mathematics teacher educators in the framework of a M.Ed. program in Pakistan (Jaworski, 2001), and professional growth of mathematics teacher educators via their practice teaching in Israel (Zaslavsky & Leikin, 2004) and in Norway (Goodchild, 2007). When considering the literature's extensive focus during the past two decades on the field of training and professional development of math teachers, it is surprising that the field of training and professional development of mathematics teacher educators receives almost no attention. Indeed, the important role attributed to the professional development of teachers in advancing the teaching of mathematics also mandates much greater attention to the subject of training and professional development of mathematics teacher educators.

Another problem that can be identified when reviewing the literature is that the field of professional development of math teachers is not defined. This is expressed, for example, in the fact that no accepted term exists in the contemporary educational discourse for someone who engages in the professional development of teachers (unlike those who engage in training prospective teachers, who are called teacher educators). Terms that are frequently used include: "mentors," "instructors," "advisors," "continuing education facilitators," "leading teachers" and so on. The lack of an agreed-upon name for those who actually engage in the professional development of teachers reflects the unclear nature of the field of professional development of math teachers – both its academic nature and practical nature – in Israel and in other countries. Indeed, one of the problems underlying the challenge of the professional development of math teachers, a problem identified by Sztajn, Ball, & McMahon (Sztajn, Ball, & McMahon, 2005), is that the group of people who actually engage in this is not clearly defined. In many countries, this group includes university lecturers and school teachers; people whose primary job is to work with teachers and those who do this only as temporary work on a part-time basis in addition to other work; those who also work with prospective teachers and those who work only with teachers. But not only is the group of people involved in the professional development of teachers insufficiently defined. Wilson and Berne (Wilson & Berne, 1999) contend that the system of professional development of math teachers is often arbitrary, sporadic and non-obligatory.

A third problem that can be identified from a review of the literature in the field of training and professional development of mathematics teacher educators is that there is a lack of information about their actual work. The literature in the field of math education offers countless ideas for planning the professional development of math teachers, with the aim of influencing the teaching and learning of mathematics in the school. However, the research

Central problems that can be identified when reviewing the literature in the field of training and professional development of mathematics teacher educators are a lack of research on the subject, an absence of clear definitions, and scant information about the practical work of teacher educators.

literature offers very little empirical information about the actual work of mathematics teacher educators. A quick survey of the articles in the leading international journal in the field of training and professional development of math teachers – the *Journal of Mathematics Teacher Education* – indicates that research in this field tends to focus on what the teachers participating in professional development programs learn and not on the nature of the work performed by those who lead such programs. Moreover, the findings of a survey conducted several years ago on research in the field of training and professional development of math teachers (Adler, Ball, Krainer, Lin, & Novotna, 2005) showed that nearly all of the empirical research in this field includes self-reporting by teacher educators about their work and therefore reflects only a tiny part of the work in this field. That is, it reflects the work performed by teacher educators who publish research articles in journals and conference publications. (Some of them are members of an academic faculty and leading scholars in the field of math education.)

A glance at the practical work of teacher educators who are not members of an academic faculty is presented in the studies of Stein et al. (Stein et al., 1999) and of Even, Robinson and Carmeli (Even, Robinson, & Carmeli, 2003). Salient characteristics of the practical work described in these studies include presenting sample lessons, analyzing the principles of a new program of studies, encouraging teachers to express their concerns and helping teachers in resolving concrete, practical problems pertaining to concerns they have about particular components of a new program of studies. The study by Stein et al. also describes the characteristics of the practical work of teacher educators who are members of an academic faculty, including confronting teachers with conflicts between new ideas and existing beliefs and practices. However, the empirical research literature does not provide information on the question of whether the practical work noted in the aforementioned research indeed characterizes the practical work of mathematics teacher educators.

7.3 What do educators of secondary school math teachers need to study?

The central role of educators of secondary school math teacher is to help the teachers learn more about the teaching of mathematics and to support the development of the teachers' knowledge and practical work. This role can be performed in a variety of ways and work environments, in which the teacher educators fill different functions and work with diverse groups of teachers in terms of education, teaching experience, socio-cultural background, motivation and so on. Examples of this include work at a regional teachers' center or pedagogic center (Pisgah), and work focused on developing a professional approach to teaching and learning mathematics as part of ongoing professional development. Another example is to offer short courses on a particular subject to a large group of teachers throughout the country. Another way is to work with a group of teachers at the school of the teacher educator, and thus expand the role of the subject coordinator to also include the professional development of the school's teachers. A different format for the work of the teacher educator is to train teachers in the framework of a project to develop and institute a new program of studies, while focusing on the teachers' professional growth. In other words, teacher educators should be trained to work in various and unexpected conditions so that based on the training they receive, they can develop ways of supporting the teachers' learning and promoting the professionalism of teachers, preparing them for the situations they will encounter and the specific roles they will need to fill, together with their fellow teachers and in accordance with their personal preferences.

Discussions pertaining to the training and professional development of teachers tend to start with questions of knowledge: What do teachers, or in our case teacher educators, need to know? However, discussions about the question of what teachers, or teacher educators, need to learn cannot only focus on knowledge because teaching, and in our case the training and professional development of teachers, is something one does and not merely something one knows. Therefore, besides identifying fields for a professional knowledge base, attention should also be devoted to the work of the teacher educators and focus on practices while combining both aspects of the professionalism of teacher educators – knowledge and practical work.

The literature points to three main fields comprising the professional knowledge base of educators of secondary school math teachers (Cooney, 1994, 2001; Even, 1999, 2005; Jaworski, 2001; Zaslavsky et al., 2003). The first two are also included in the knowledge base required of secondary school math teachers in both formal mathematics and math literacy. But the third field is unique to teacher educators:

★ Math education – the teaching and learning of mathematics
★ Mathematics – of secondary schools and advanced mathematics
★ Training and professional development of teachers – professional development and learning of teachers

In addition to the focus on developing knowledge about the learning and professional training of teachers, the teacher educator must study the practical aspects of the work of teacher educators:

★ Practices for professional development of teachers – methods of training and professional development of teachers

In addition to the focus on developing knowledge about the learning and professional training of teachers, the teacher educators should also study the practical aspects of the work of teacher educators. The research indicates that teachers, who are generally the ones performing the role of teacher educators, should study such practices.

These methods include, for example, the practical work of planning, running and evaluating activities, seminars and courses for math teachers, as well as decision-making practices pertaining to the objectives of training and professional development activities for math teachers, reflection and professional discussions with colleagues. The research indicates that teachers, who are generally the ones performing the role of teacher educators, should study such practices (Even, 1999, 2005). This aspect of learning the practical work of mathematics teacher educators is particularly important because, as already noted above, support for teachers' learning is not a simple matter, and sometimes activities for the professional development of teachers do not promote teachers' learning that improves the actual teaching in the classroom (Borko, 2004; Wilson & Berne, 1999; Zaslavsky, Chapman & Leikin, 2003).

7.4 How should teacher educators of secondary school math teachers study?

Research on math teachers' learning in various frameworks of training and professional development indicates that in the teachers' learning process there is considerable importance in questioning teachers about their practices and reflection by teachers about their work (Ball & Cohen, 1999; Jaworski, 1998; Krainer, 1998; Loucks-Horsley et al., 1998; Zaslavsky et al., 2003). Past experience shows that these aspects are also important to those who are studying to be mathematics teacher educators (Even, 2005). However, we currently lack conceptual-theoretical frameworks that could serve as a solid foundation for shaping the learning experiences of mathematics teacher educators. Initial attempts (Even, 2005) show some potential for developing learning experiences for mathematics teacher educators, experiences derived from conceptual-theoretical frameworks in the field of learning mathematics that combine cognitive and socio-cultural approaches (Collins, Brown, & Newman, 1989; Yackel & Cobb, 1996) – together with theoretical frameworks for learning practical work. An example of this is "situated learning," which places emphasis on learning practices in the field of work by gradually participating in this field through "legitimate peripheral participation" (Lave & Wenger, 1991) – a process in which learners become full participants in a community's socio-cultural practices.

7.5 Recommendations

> Current efforts to improve the training and professional development of secondary school math teachers are being weakened by the absence of proper training for those who fill the roles of mathematics teacher

educators and by failing to foster this group as a professional group. On the other hand, the research literature provides little information on the subject of training mathematics teacher educators. Therefore:

A. The committee recommends that academic institutions develop and operate programs for training educators for secondary school math teachers, based on the principles outlined above.[38]

B. The committee recommends offering paths for continuing the professional development of mathematics teacher educators, while building and reinforcing their professional identity.

C. The committee recommends complementing the activity with research that can provide useful feedback for improving the training, continuing the professional development and building a professional group.

D. The committee recommends developing dedicated training materials for educators of secondary school math teachers.

[38] Subject coordinators should be encouraged to undergo training as teacher educators, thus strengthening their ability to guide the math teachers in their school.

Chapter 8
Evaluating Teachers' Performance – Managing the Quality of Teaching

8.1 Evaluation for improvement and evaluation for advancement

The evaluation of teachers during the course of their career is aimed at improving the teachers' performance (formative evaluation that leads to professional development), determining their advancement in the accepted hierarchy of ranks in the profession and demonstrating accountability (summative evaluation).

A. Evaluation for improvement

The evaluation of teachers for purposes of improving their teaching is usually internal and takes into account the context in which the teacher is employed. The chance that formative evaluation fulfills its objective increases when it occurs in conditions such as (OECD, 2005):

1. A non-threatening context
2. An organizational culture of reciprocity in giving and receiving feedback
3. Clear and shared individual and collective goals regarding the improvement of teaching in the school
4. Simple evaluation tools such as tools for self-assessment, classroom observation and structured interviews
5. Supportive school leadership
6. Opportunities for professional development and the availability of resources and means for improving the teacher's activity

7. Integrating the evaluation of teachers in a general array of evaluation and assurance of school quality

B. Evaluation for accountability and advancement

To ensure the validity and effectiveness of the evaluation of the teacher's performance for the purpose of accountability (summative evaluation) and for the purpose of advancement, it should meet the following conditions (OECD, 2005):

1. It should independently and objectively assess the teacher's performance
2. It should be based on national standards and criteria
3. It should include a component of evaluation that is external to the school and should be based on formal processes of evaluation
4. It should be based on well-established rules regarding the repercussions of the evaluation
5. It should address primary individual objectives in all aspects of the teacher's performance
6. It should be conducted by evaluators who have appropriate training and qualifications
7. It should use the results of the evaluation as the basis for a program of professional development
8. It should allow the possibility of appeal for teachers who feel they were unfairly treated (OECD, 2005).

8.2 The responsibility for evaluating teachers

The role of the education authorities: The education authorities play an important role in conceptualizing and implementing the evaluation of teachers in general and the evaluation of math teachers in particular. These authorities set the national goals for learning output and the accepted standards for the teaching profession, and establish norms that structure (regulate) the assessment of teachers. In some countries, the education authorities play a direct role in monitoring and running the evaluation procedures. Their role includes planning specific evaluation tools, defining criteria for evaluation, assigning evaluation duties and follow-up of the evaluation results. In other countries, the education authorities define only general guidelines and principles, leaving the schools broad freedom to adapt the model of teacher evaluation to their needs (for example, enabling the school to define the criteria of the evaluation).

The role of supervisors: In many countries, the supervisors assume full responsibility for evaluating teachers – a responsibility that includes developing assessment mechanisms and evaluating individual teachers. In other words, the supervisors fill the role of evaluators. In other countries, they are not responsible for evaluating the individual teacher, but they fill an important role in driving the process of enhancing the quality of the school's leadership and the quality of teaching. In general, this is done by combining the results of the school's external evaluation with the results of feedback on the leadership and administration, feedback on the quality of the teaching and learning processes, and feedback on the school atmosphere. The supervisors usually play a substantial role in modeling and disseminating successful activity in evaluating teachers, as part of a quality control mechanism for the entire school system.

The role of the school leadership: Effective implementation of teacher evaluation largely depends on the way the concept is instituted and the actions of the school leadership. The school leadership plays different roles in the evaluation of teachers in different countries. In some countries, this role amounts to administering procedures dictated from above. In such cases, real leadership and pedagogic awareness is required in order to utilize the process of teacher evaluation as a catalyst for development and not merely as a bureaucratic instrument. In other countries, school principals take upon themselves full responsibility for evaluating teachers. In Finland, for example, the school principal – who is the pedagogic leader – is responsible for the teachers, the school and the use of indexes that are required for improving the quality of teaching. Consequently, in most of Finland's schools there is a system of annual discussions focusing on evaluating the achievement of the individual goals the teacher set during the previous year, and on defining the teacher's developmental needs for the following year (Larsen, 2006; Eurydice, 2004). Due to the complex connection between developing personnel, career development and school development, it is essential that the processes of evaluation, administration and improvement be part of the school's overall quality assurance and control.

In Israel, school principals bear responsibility for evaluating teachers and providing feedback, in addition to self-assessment by the teacher.

However, in secondary education, which is characterized by professional teaching, it seems difficult to expect a principal to effectively evaluate a professional teacher if the principal is not educated in the particular field of knowledge or in the teaching of it.

The role of teachers: In some education systems, the evaluation of teachers is based on reviews by peers who are usually more experienced and in positions of greater responsibility. Peer reviews are used more for purposes of improvement. There is evidence that observations conducted by colleagues for purposes of improvement indeed contribute greatly to the quality of teaching. When peers serve as evaluators in accountability-driven procedures, questions of ethics and legitimacy arise that require a valid response (OECD, 2005).

In Israel, school principals bear responsibility for evaluating teachers and providing feedback, in addition to self-assessment by the teacher. The evaluation is conducted with a comprehensive assessment tool developed by the National Authority for Measurement and Evaluation (RAMA), and which now operates in elementary schools and in some of the middle schools in the framework of the *Ofek Hadash* agreement. According to the *Oz Letmura* agreement, a similar evaluation is slated to begin in secondary education during the 2012-2013 school year. However, in secondary education, which is characterized by professional teaching, it seems difficult to expect a principal to effectively evaluate a professional teacher if the principal is not educated in the particular field of knowledge or in the teaching of it.[39]

[39] Detailed information about the assessment tool and the evaluation process is available on the website (Hebrew): http://cms.education.gov.il/EducationCMS/Units/Rama/Haarachat_Morim/Haarachat_Morim.htm

8.3 The evaluation framework: Connection with other school dimensions

Evaluation of the individual teacher and collective responsibility for the quality of teaching in the school

The systems of school evaluation, teacher evaluation and feedback all share the goals of maintaining standards and improving the students' performance. Therefore, it appears that synergy (active cooperation) between the school evaluation and the teacher evaluation is very beneficial. In order to achieve the maximal effect, it is essential to link the focus of the school evaluation to the focus of the teacher evaluation (OECD, 2005).

The systems of school evaluation, teacher evaluation and feedback all share the goals of maintaining standards and improving the students' performance. Therefore, it appears that synergy (active cooperation) between the school evaluation and the teacher evaluation is very beneficial.

There are different ways to link the evaluation of teachers to school arrangements, with the goal of improving teaching. First, when the evaluation of teachers is conducted as part of an internal school evaluation, separate from a formal and individual evaluation of teachers, its direct aim could be to enhance the teaching process in the school. The evaluation of teachers for improvement as part of the internal school evaluation is conducted by the principal or by the administrative team and the team of peers. The results of this type of evaluation can help to identify teaching needs and contribute to defining a school plan whose objectives include improving the teaching process in the school.

Secondly, when the contribution of the individual teacher is weighed in the framework of his or her evaluation – the school processes and the teacher evaluation will be intertwined. In some countries, teachers are evaluated relative to their responsibility for their students' learning and for their contribution as part of the school's faculty. Areas of evaluation include the teachers' professional responsibility – professional development, contribution to administering the school, involvement in the community, and so on.

Thirdly, when professional development activities, based on the evaluation of the individual teacher, are planned in the context of the general plan for developing the school – the school proceedings and the teacher evaluation will be linked. In some countries, professional development is now included in the school's development priorities, and the actual professional development of the teachers is channeled accordingly (OECD, 2005).

Using teacher evaluation results in evaluating the school

Only rarely are the results of the evaluation of individual teachers used for an external evaluation of the school. The evaluation of teachers generally has a limited focus because it is intended to identify priorities for the professional development of the individual teacher. The use of teacher evaluation is more prevalent in the context of an internal evaluation in the school. In some countries, these evaluations are taken into account when evaluating the quality of teaching in the school. That is, the evaluation of teachers can help in developing school improvement programs (OECD, 2005).

The connection between school evaluation, teacher evaluation and student evaluation

It is possible to link the evaluation of the schools, the evaluation of the teachers and the evaluation of the students. The students' scores in standard national tests are considered a basis for judging the school's performance, and indirectly for evaluating the teaching staff (for example, in Sweden and Scotland). The current transition from a process-based evaluation of teachers to an output-based evaluation of teachers is a step in closing the gap between individual responsibility and collective accountability. In a large number of countries, the mechanisms of supervision treat teachers as both individuals and as members of the school staff (Ellet, & Teddlie, 2003).

8.4 Teacher evaluation procedures

The institutionalization of effective procedures for evaluating teachers is a challenging mission on various levels: accuracy of assessment, inclusion of all of the dimensions that are slated for assessment, consistency in feedback, adaptation to the needs of the stakeholders (teachers, school leaders, education authorities), cost-benefit and practical feasibility.

The institutionalization of effective procedures for evaluating teachers is a challenging mission on various levels: accuracy of assessment, inclusion of all of the dimensions that are slated for assessment, consistency in feedback, adaptation to the needs of the interested parties (teachers, school leaders, education authorities), cost-benefit and practical feasibility.

The evaluation of teachers requires the establishment of standards and criteria for evaluation, conditions for suitable assessment of performance (method of operation and sources of information). The use of students' scores for evaluating the individual teacher (value-added evaluation) is an example of a particularly challenging use because the learning output is a product of many factors, including the student's background variables and socio-economic situation.

Furthermore, teachers have a special need for feedback. Therefore, evaluation processes must be planned that answer this need. The improvement of the effectiveness of teacher evaluation means choosing the appropriate evaluation processes for given objectives (for example, an internal versus external process, and a formative versus summative process).

The nature and essence of teacher evaluation

1. **The evaluated teachers:** A particular model of evaluation can include all of the teachers in the system or subsystem that is defined by the type of employment (permanent or contract), by career stage (beginning versus experienced), by education rank (elementary or secondary school) or by type of education (general, vocational, adult education, and so on).

2. **The nature of the evaluation:** The evaluation can be mandatory or optional. Optional evaluations can be linked to promotion during the teacher's career.

3. **The essence of the evaluation:** The evaluation can be external or internal. In the case of external evaluations, the aspects that are evaluated, the tools used and the criteria are common to all of the schools, and the evaluators are usually from outside of the school in which the evaluated teachers are employed. In the case of internal evaluations, the school assumes responsibility for planning specific tools and criteria for assessing and monitoring the evaluation results, and the evaluators are usually from within the school.

4. **Frequency:** The evaluation of teachers can be conducted at regular intervals (for example, every two years), during the initial stages of a teacher's career (for example, when a teacher is under consideration for promotion), or at milestones during the course of the teacher's work (for example, after a trial period or when renewing a contract).

School-based evaluation of teachers versus external evaluation of teachers

In schools characterized by evaluation systems that are not fully developed, the internal evaluation is likely to suffer from the school's lack of experience in developing assessment tools and in evaluating teacher performance. In the case of evaluation for the purpose of accountability, there is justification for using a national evaluation framework and standard procedures.

An in-school evaluation of teachers offers the advantage of giving the school control over the evaluation processes and ensures that the school carefully considers all aspects of the evaluation. This evaluation also ensures that the school context is taken into consideration: The teacher is evaluated according to standards and criteria that are consistent with the school's goals and its socio-educational background. However, in schools characterized by evaluation systems that are not fully developed, the internal evaluation is likely to suffer from the school's lack of experience in developing assessment tools and in evaluating teacher performance. Moreover, the use of assessment tools developed by the school itself is more suitable for evaluation aimed at improvement. In the case of evaluation for the purpose of accountability, there is justification for using a national evaluation framework and standard procedures, particularly when the repercussions of this type of evaluation are at the national level and there is a danger that different standards will be used by different schools.

8.5 Standards (for reference or comparison), evaluated aspects and criteria for evaluation

Standards (for reference or comparison)

In order for the model to be fair and credible, standards for comparison are required. A set of standards is essential in order to evaluate the teachers relative to what is considered "good" teaching. Teaching abilities and types of responsibility of teachers must be defined in order to formulate a comprehensive definition of what the teachers should know and be able to do in the framework of their job. In general, the standards for evaluating teachers are:

1. The professional profiles of the teacher (a general profile of the teacher's abilities), including special profiles for different types of teachers (for example, education rank, subject of instruction).
2. A series of general and professional duties of the teacher, including descriptions of the job.
3. At the school level – the school development plan, the internal rules and the annual plan of activity.

A key component in the professional profile of the teacher is a clear and precise description of what the teacher is expected to know and be able to do. A prerequisite for preparing a profile of the teacher's abilities is a description of the goals for students' learning. The teachers' work, and the knowledge and the skills they need in order to be effective, should reflect the learning

A key component in the professional profile of the teacher is a clear and precise description of what the teacher is expected to know and be able to do. A prerequisite for preparing a profile of the teacher's abilities is a description of the goals of the students' learning. The teachers' work, the knowledge and the skills they need in order to be effective should reflect the goals of the students' learning.

goals the school seeks to achieve (fulfill). It is essential to have relevant standards for the profession as a whole (profession-wide) and a shared understanding of what is considered suitable teaching (OECD, 2005).

The teachers' profiles often reflect different levels of performance, corresponding to beginning teachers, experienced teachers and teachers who fill senior positions. It is important to note that the professional profile provides a common basis for organizing the key components of the teaching profession: teacher training, the licensing of teachers (entrance exams, for example), teachers' professional development, career advancement, teacher evaluation and so on.

Evaluated aspects

The standards were established for main areas of evaluation. An important contribution in this area is "the framework of teaching" proposed by Danielson (Danielson, 1996), a framework formulated in order to provide a "road map" to guide beginning teachers in their initial experiences in the classroom, while helping experienced teachers to become more effective and serving as a means to focus improvement efforts.

An important contribution in this area is "the framework of teaching" proposed by Danielson (Danielson, 1996), a framework formulated in order to provide a "road map" to guide teachers.

The framework categorizes the teachers' roles and duties into four main areas:

1. **Planning and preparation:** Demonstrating knowledge in content and pedagogy, demonstrating knowledge about the students, choosing

teaching objectives, planning coherent instruction, evaluating the students' learning.

2. **The classroom atmosphere:** Creating an atmosphere of respect and trust, establishing a culture of learning, managing class procedures, managing student behavior and organizing a physical environment.

3. **Teaching:** Clear and precise communication, use of questioning and discussion techniques, involving students in learning, providing feedback to students, demonstrating flexibility and responsiveness (reactivity).

4. **Professional duties:** Reflection of teaching, management of accurate records, management of communication with the parents, contribution to the school and to the district, professional development, demonstrating professionalism.

Each of the components includes some elements for evaluation. For example: the teacher's knowledge about the students includes elements such as knowledge about the characteristics of the age group, knowledge about the students' various learning approaches, and so on. This framework has influenced a large number of evaluation systems in the world.

Evaluation criteria

The important basis for conducting evaluation is to maintain clear criteria for using evaluators who are suitably trained.

The important basis for conducting evaluation is to maintain clear criteria for using evaluators who are suitably trained. This requires formulating clear definitions of what is expected of the professional activity. Procedures for evaluating teachers demand the definition of criteria for determining the level of the individual teacher's performance in each of the evaluated aspects. This means developing standard criteria or formulas for documenting the teacher's performance. An additional criterion is the weighing of the various evaluated aspects and the calculation of an overall quantitative ranking, if such a ranking is indeed part of the model of teacher evaluation. The quantitative ranking can later be tied to a qualitative scale – for example, "unsatisfactory," "basic," "skilled-expert," "outstanding."

Some of the models of teacher evaluation define a particular percentage of teachers who can be ranked at the top of the scale.

Some of the models of teacher evaluation define a particular percentage of teachers who can be ranked at the top of the scale (that is, "outstanding"). This approach is contrary to the criterion-based approach, but its use can be justified in cases of an emerging society and a tradition of evaluation models aimed at preventing a situation in which most of the teachers are ranked at the top of the scale. In such cases, the scale loses its value, of course.

8.6 Recommendations

The committee recommends primarily maintaining a routine of formative evaluation for secondary school math teachers, but it does not reject conducting a summative evaluation at various stages of the teaching career.

Evaluation for the purpose of improvement:

1. Should be conducted at varying intervals, for example according to teaching experience or according to the previous feedback results.

2. Should be concrete, should address positive aspects as well as the aspects that need improvement, and should be timed in a way that enables real-time implementation of the evaluation results.

3. Should be performed by someone trained in evaluation and who is knowledgeable in mathematics and in math education. The mathematics coordinator, the district or national instructor of math teaching, the director or deputy director can perform this mission if they meet the conditions cited above.

4. Should initially be conducted with existing tools, such as the teacher evaluation tool developed by National Authority for Measurement and Evaluation (RAMA).[40]

Evaluation for the purpose of accountability and promotion:

1. Should be performed with the involvement of an external entity in the evaluation.

2. Should be based on uniform and transparent criteria, including special activity and unique contribution to the school and community.

3. Must involve the teachers in the evaluation results and offer teachers an option to appeal if they believe they did not receive a fair evaluation.

[40] The tool developed by RAMA includes four main indexes for evaluating teachers: perception of the teacher's role and professional ethics, the field of knowledge, learning and educational processes, and partnership in a professional community. The committee believes that it is a comprehensive and successful tool, but in all of the indexes the evaluation in secondary school education should be performed by someone with knowledge and experience in both the field of knowledge and the field of evaluation.

Chapter 9

Environmental components supporting the full utilization of the teachers' knowledge

Besides describing the knowledge required for those engaged in teaching mathematics in secondary schools, the committee deemed it appropriate to also cite several factors in the teachers' environment that are likely to help them in their work and influence their ability to bring their knowledge into expression.

The teachers' environment is an important topic that includes numerous and diverse components, and deserves a more detailed discussion that goes beyond the committee's primary mandate.[41] However, the committee did not want to ignore this completely and chose to briefly note three environmental factors that it believes are likely to contribute to the work of math teachers and improve the knowledge of their students.

1. Textbooks
2. Technological and other teaching aids
3. A community of teachers

[41] For example, the OECD organization recently developed the TALIS (Teaching and Learning International Survey) exam, which was administered for the first time in 2008, and will be administered again in 2013. The exam is designed to test environmental components that support teaching in the middle schools and focuses on the teachers' professional environment, the teaching conditions and their impact on the effectiveness of the teachers and schools. http://www.oecd.org/document/40/0,3746,en_2649_39263231_47766184_1_1_1_1,00.html

1. Textbooks

Textbooks in mathematics for middle schools are in a trial stage, pending approval by the Ministry of Education, in accordance with the program of studies. On the other hand, there is no approved program of studies in high school and the books the Ministry of Education approved in the past for use in high school no longer match the content in the matriculation exams. In the absence of relevant approved books, teachers use books that do not have the official approval of the Ministry of Education,[42] and these generally include many exercises and minimal explanations. This leads to the fact that students are not accustomed to learning on their own from the books and experience difficulty in reading mathematical texts. A student who misses a lesson cannot make up the material by learning only from the book.

The committee believes that there is an urgent need to approve a program of studies for high school, to develop relevant textbooks for this program, in Hebrew and in Arabic, and to approve them – as in the process the middle schools underwent. The committee believes that textbooks approved by the Ministry of Education must include the development of ideas, presentation of concepts and examples, discussion of work strategies, familiarity with various tools for mathematical activity, and suggestions for their use. Such textbooks are an essential resource for teachers, helping them to implement the program of studies, to organize teaching times, and to emphasize the main points. A resource like this will enable teachers to refer the students to study on their own when necessary, and thus also develop their skills in reading texts and learning independently.

Textbooks must include the development of ideas, presentation of concepts and examples, a discussion of work strategies, familiarity with various tools for mathematical activity, and suggestions for their use.

[42] According to Directive 5772/8 (April 1,2012) from the director-general of the Ministry of Education, schools are required, starting from the 2012-2013 school year, to select textbooks from the ministry's list of approved textbooks. Until a program of studies for high schools is approved, temporary permission is granted to use the widely used textbooks in mathematics for one year only.

In addition to developing textbooks, there should be an effort to encourage the writing of books intended only for the teacher and which are not linked to a particular textbook, which are based on all of the desired components of knowledge described in Chapter 3 of this document, and which would serve as advisors to the teachers.

In order for a textbook to gain approval from the Ministry of Education, it must be accompanied by a teacher's manual. Such a manual is necessary but not always sufficient. The committee believes that in addition to developing textbooks, there should be an effort to encourage the writing of books intended only for the teacher and which are not linked to a particular textbook. Such books, based on all of the desired components of knowledge described in Chapter 3 of this document, would serve as close advisors to the teachers in their work: They would provide an explanation and the goal and nature of each subject, including its main ideas, distinguishing between the important and unimportant aspects of each subject, building connections between various subjects (including subjects from other fields), a description of the mathematical horizon of each subject, a description of special didactic aids for each subject, a description of the typical difficulties of students, examples of regular and special problems, a presentation of alternative approaches and solutions for mathematical problems, and more. Such books, or a separate book designed for this purpose, should also support the teacher's work in a heterogeneous classroom.

2. Technological and other teaching aids

Various surveys conducted in the world indicate that despite the research and development on applications of technology in mathematics education, there has been very little use of these applications in the classroom relative to other subjects (see for example Sutherland, Clark-Wilson & Oldknow, 2011). The committee recommends that in parallel to equipping schools with display technologies, Ministry of Education resources will be invested in helping to equip schools with technological learning environments aimed at learning mathematics (such as dynamic environments for learning geometry, environments for learning functions, environments for statistical research, and more). Resources should also be directed toward developing appropriate learning materials for teaching and learning in these environments, for disseminating them to teachers at no cost, and for dedicated frameworks of professional development for supporting the use of computers in teaching mathematics.

In parallel to equipping schools with display technologies, Ministry of Education resources should be invested in helping to equip schools with technological learning environments aimed at learning mathematics and developing appropriate learning materials for teaching and learning in these environments, for disseminating them to teachers at no cost, and for dedicated frameworks of professional development for supporting the use of computers in teaching mathematics.

In order to support the varied and creative teaching of mathematics, the committee also recommends reinforcing the use of simple teaching aids. For this purpose, the committee recommends that there be a special mathematics classroom in each school with teaching aids such as rulers and compasses, geometrical objects, software, computer-based training programs, videos, a projector, a computer, and a smart screen.

In addition to these, it is important that this room include a library with a collection of math books and journals (virtual and printed), books on the history of mathematics, books on problems on particular subjects (including subjects that are only learned in enrichment classes), and so on. This type of library can serve as a source of teaching ideas for the teacher and as a center for referring interested students to expand their knowledge beyond what is learned in the classroom. In the library, the teacher can find various mathematical problems for enriching the students. The library should also contain books on the history of mathematics that expand the student's horizons.

3. A community of teachers

The committee recommends supporting various steps to establish and encourage the creative work, activity and visibility of teacher communities.

As in every professional community, the support of colleagues and reciprocal learning among the members are likely to contribute to improving the teachers' work (see for example Okur & Bahar, 2010; Vassiliou, 2011), help them contend with the sense of being alone, empower the mutual enrichment and sense of belonging to the community, and even spur the development of "team spirit." This is true for all of the teachers, and even more so for beginning teachers. The committee recommends supporting various steps to establish and encourage the creative work, activity and visibility of teacher communities. In particular, the committee recommends the following frameworks:

A. Regular meetings of the staff of math teachers in the school (to discuss shared problems, to jointly prepare evaluation tools, to discuss various approaches to a particular subject, to exchange information on resources that have proven to be effective, and so on). One of the teachers should coordinate this activity, set the agenda, monitor innovations in the teaching of mathematics in the literature and present them for discussion at the staff meetings.

B. Periodic meetings with teams of science teachers, art teachers and teachers from other fields (for coordination, exchange of information, discussion of common subjects, clarification of the use of concepts from one field in other fields, and so on) – for example, collaboration with art teachers (for the purpose of clarifying concepts such as perspective and the golden section, and for developing models for three-dimensional views) is like to be particularly fruitful.

C. Conducting a regular array of training by the Ministry of Education aimed at boosting the sense of belonging to the national professional community. In this context, it is recommended to strengthen the professional conference for math teachers: The conference helps to boost the status of teachers and also contributes to their knowledge and achievements. Continued support should be given to organizing the conferences by the Teachers' Center and providing possibilities for teachers to sound their voices, to express their views and to conduct a profession discourse with their colleagues, with mathematicians and with math educators.

It is recommended to strengthen the professional conference for math teachers and it is very important to continue to develop the Internet site and to use it to increase internal communication within the community of teachers and to transform it from a tool that accompanies the learning process into an active partner in the field of teaching mathematics in general.

D. Continuing and even increasing support for the existing Internet site at the National Center for Secondary School Math Teachers. The site is a rich resource that supports the professional development of teachers and the formation of a community of teachers. It is very important to continue to develop the Internet site and to use it to increase internal communication within the community of teachers and between the community of teachers and other professional communities. Today it is clear that the Internet is an active tool accompanying the teaching of mathematics in Israel and abroad, and that due to its great potential it should be transformed from a tool that accompanies the learning process into an active partner in the field of teaching mathematics in general. The aim should be to develop the site to become the "canon" and to make it into the leading math site in Israel (in the same way that Mikra-Net is the leading site for teaching the Bible). This is a welcome and substantial national mission.

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Appendix
Committee Members' Brief Biographies

Prof. Hanokh Gutfreund, professor at the Hebrew University of Jerusalem; since 1985, has held the Andre Aisenstadt Chair in theoretical physics. He has previously held various academic and administrative positions at the University – head of the Physics Institute, head of the Advanced Studies Institute, rector and president. He was among the initiators and founders of the Center for Neural Computation and is currently a member of the Center where he also teaches.
Prof. Gutfreund is the director of the Einstein Center and is the University's appointee responsible for Albert Einstein's intellectual property; he heads the executive committee of the Israel Science Foundation and is the chairperson of the "Basha'ar – Academic Community for Israeli Society" association.
Prof. Gutfreund received his Ph.D. in theoretical physics in 1966 from the Hebrew University of Jerusalem.

Prof. Abraham Arcavi, professor in the Department of Science Teaching, which he headed from 2001 to 2005, at the Weizmann Institute of Science. His research focuses on teaching and learning mathematics in middle and secondary school. His post-doctoral work was carried out at the University of California, Berkeley.
Dr. Arcavi earned his Ph.D. in mathematics education from the Weizmann Institute in 1986.

Dr. Meir Buzaglo, lecturer in the Department of Philosophy and director of the Institute for Innovation in Education at the Hebrew University of Jerusalem. The areas he researches include the philosophy of mathematics, the philosophy of physics, the philosophy of language and the philosophy of Judaism.

Dr. Buzaglo is involved in the field of mathematics education – he founded the "Wellsprings of Education" institute for teaching mathematics in poor neighborhoods and development towns and served as a teacher, researcher and developer of curricula for teaching mathematics and logic.
He has an M.A. in mathematics and a Ph.D. in philosophy, both from the Hebrew University of Jerusalem.

Prof. Ruhama Even, professor at the Weizmann Institute of Science, heads the Mathematics Group in the Department of Science Teaching and directs the Teaching Certification program at Weizmann's Feinberg Graduate School.
Her main research interests include math education, education and professional development for math teachers and teacher educators, and the interactions between the math curriculum, teachers, and classrooms.
Prof. Even earned her Ph.D. in Mathematics Education from Michigan State University in 1989.

Dr. Hagar Gal, senior lecturer and head of the Mathematics-Physics Department at the David Yellin College of Education. Dr. Gal was formerly a teacher of mathematics and an education mentor; she also founded and directed the Regional (Comprehensive) High School in Ma'ale Efraim, developed and taught in-service education and professional development programs for math teachers, and at the David Yellin College, initiated and heads the "From One End to the Other" program, for the instruction of mathematically and scientifically gifted students in mixed-ability classes.
Her areas of expertise include professional training and development for mathematics teachers (with an emphasis on teaching students that find math difficult on the one hand, and on mathematically talented students, on the other) and on wide-ranging aspects of the difficulty in teaching geometry.
Dr. Gal holds an M.Sc. in applied mathematics (1978), a teaching certificate for mathematics and physics and a Ph.D. in mathematics education from the Hebrew University of Jerusalem (2005).

Dr. Bella Kessler, mathematics teacher (retired), taught for over 20 years at the Hebrew University High School and at the David Yellin College of Education, and was a regular participant in the "Songs and Sessions" radio program.

Following her retirement, she directed the Israeli School in Moscow; currently, she is involved in training elementary school mathematics teachers and in the Mathematics for Outstanding Students project in the School of Education at the Hebrew University.
Dr. Kessler has a teaching certificate in mathematics from the University of Kishinev and a Ph.D. (applied mathematics) from the Moscow Institute of Economics and Statistics, received in 1973.

Prof. Raz Kupferman, professor at the Einstein Institute for Mathematics at the Hebrew University of Jerusalem. He carried out his post-doctoral research at the University of California, Berkeley; his current research focuses on scientific computation, flow of complex fluids, elasticity, stochastic modeling and bio-mathematics. Prof. Kupferman is also involved in the field of mathematics education.
He holds a Ph.D. in physics from Tel Aviv University, received in 1995.

Prof. **Roza Leikin,** professor in the mathematics education program, heads the M.A. program in Education of Gifted and Talented in the education faculty's Department of Counseling and Human Development, and directs the Interdisciplinary Center for Research and Advancement of Giftedness and Excellence (RANGE Center) at the University of Haifa. Her areas of expertise include mathematics teachers' knowledge and professional development, mathematics challenges in education, giftedness and creativity in mathematics.
Dr. Leikin earned her D.Sc. in 1997 in mathematics education (teacher education) from the department of science education at the Technion.

Prof. Varda Liberman, deputy dean, Arison School of Business at the Interdisciplinary Center (IDC), Herzliya, director of mathematical and statistical studies at the IDC and a visiting researcher at Stanford University.
Prof. Liberman's research and writing focuses on probabilistic judgments and decision making. She previously headed the department of mathematical education at the Open University and wrote many text books on the subjects of mathematics and statistics. She created a unique curriculum for developing probabilistic thinking, authorized by the Ministry of Education's professional committee to be included as part of the mathematics curriculum, and wrote

(together with the late Prof. Amos Tversky) a textbook on the subject, entitled "Probability Thinking in Everyday Life."
Prof. Lieberman holds a Ph.D. in mathematics from Tel Aviv University, received in 1985.

Ms. Ifat Nachshon, mathematics teacher at the Nehalim Torah Technology High School and a Davidson Institute for Science Education mentor in the Improving Mathematics Learning program at the "Amal Ramle" interdisciplinary school, conducted in cooperation with the Weizmann Institute's Department of Science Teaching. Ms. Nachshon was previously an information systems engineer at Tadiran, Tel Aviv University, and at Tel Hashomer Hospital.
She holds a B.S. from the Technion (in information system engineering – Faculty of Industrial Engineering and Management), received in 1990, a teaching certificate in mathematics education from Shaanan College and is currently studying for her M.A. in science education in the "Caesarea Program" for outstanding teachers at the Weizmann Institute of Science.

Prof. Fadia Nasser-Abu Alhija, professor at the School of Education of Tel Aviv University, where she heads the Department of Curriculum Planning and Instruction and the Program for Research, Measurement and Evaluation Methods. Previously, she was research coordinator for GRE testing at the Educational Testing Service (ETS) in Princeton, NJ (USA). Her main research topics are measurement and evaluation of gender- and culture-related achievements; evaluation of teachers and teaching; and, the structural validity of testing methods.
Prof. Nasser-Abu Alhija earned her Ph.D. in Research, Evaluation, Measurement and Statistical Methods from the University of Georgia (U.S.) in 1997.

Prof. Anna Sfard, professor, head of the Department of Mathematics Education, Faculty of Education, University of Haifa. Until recently, she was a professor of mathematics education at the University of Michigan (Lappan-Phillips-Fitzgerald Professor of Mathematics Education) and a visiting professor at the Institute of Education, University of London. Her research focuses upon development of mathematical thinking, study of the discourse approach to thinking, and theories of learning.

Prof. Sfard holds a Ph.D. in teaching mathematics from the Hebrew University of Jerusalem, received in 1989.

Yehoshua Rosenberg (study director), teacher of mathematics and physics at a number of secondary schools; in the past, taught mathematics at the Hebrew University of Jerusalem. He holds a teaching certificate from the Lifshitz College in Jerusalem, received in 1998 and a B.S. in physics, received in 2001 from the Hebrew University of Jerusalem. He is currently completing his Ph.D. at the Hebrew University's Interdisciplinary Center for Neural Computation.